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DURATION OF QUASI-COMPLANAR ASTEROIDS REGULAR PROXIMITIES

(Presented at the 7th Meeting, held on September 26, 1980, by *R. Kašanin* and *T. Anđelić*)

The „kinematic" duration of proximity of an asteroid pair, the minimum (proximity) distance of which is ρ_p, has been defined by J. Lazović and M. Kuzmanoski in [1] as time interval around the instant t_p of actual proximity, during which their mutual distance ρ, $\rho_p \leq \rho \leq \rho_f$, ρ_f being some maximum distance chosen beforehand. The same authors adopted in [2] $\rho_f = 0.0004$ a. u., with regard to such possible amounts of their mutual perturbation as could be observable from the Earth.

In the above mentioned paper [1] the authors derived the kinematic duration of proximity for 77 quasi-complanar asteroid pairs, listed in [2], by rigorous calculus, which brought to evidence the inequality in durations of the portions of proximities preceding and following the instant t_p.

In [3] the present author defined the so-called regular proximity of an asteroid pair as the one for which the relative position vector $\vec{\rho} = \mathbf{r}_i - \mathbf{r}$ can be represented by

$$\vec{\rho} = \vec{\rho}_p + \mathbf{V}_p \tau, \qquad \mathbf{V} = d\vec{\rho}/d\tau = \mathbf{v}_i - \mathbf{v}, \qquad \tau = k(t - t_p) \tag{1}$$

within the whole time interval around t_p, of interest for the calculus of mutual perturbations. This linear dependence is made possible by the most often short duration of the phenomenon and small ρ_p. Hence, for the kinematic duration of the regular proximities from (1) and the condition of proximity $(\vec{\rho}_p\ \mathbf{V}_p) = 0$ we derive:

$$2\tau = 2(\rho_f^2 - \rho_p^2)^{1/2}\ V_p^{-1}. \tag{2}$$

Table I summarizes the first twenty quasi-complanar asteroids pairs with the smallest proximity distances, the kinematic elements of their proximities $\rho_p = |\vec{\rho}_p|$, $V_p = |\mathbf{V}_p|$, along with the kinematic durations 2τ of the phenomenon, calculated according to (2), $\rho_f = 0.0004$ a. u. The comparison with the accurate values in [1] shows a very good accordance. The differences do not exceed 0.0003 mean days, except for the „slow" proximity of the pair (960) Birgit — (1818) 1939 PE, where the difference attains 0.0028 mean days.

Table I

No.	Asteroids pair				$10^8\rho_p$	V_p	2τ
1	(215)	Oenone	(1851)	1950 VA	361	0.0811 9603	0.5727
2	(227)	Philosophia	(1737)	Severny	737	0.1025 0225	0.4536
3	(389)	Industria	(972)	Cohnia	904	0.1670 4938	0.2783
4	(960)	Birgit	(1818)	1939 PE	1024	0.0148 6944	3.1266
5	(763)	Cupido	(985)	Rosina	1399	0.1060 9106	0.4381
6	(703)	Noemi	(1130)	Skuld	1434	0.1178 0079	0.3495
7	(1736)	Floirac	(1759)	1942 RF	2202	0.1359 8398	0.3415
8	(205)	Martha	(992)	Swasey	3462	0.0578 5146	0.8009
9	(938)	Chlosinde	(1815)	1932 CE_1	4310	0.1855 0451	0.2492
10	(954)	Li	(1898)	Cowell	5068	0.1392 5801	0.3313
11	(110)	Lydia	(1393)	Sofala	6327	0.0616 1803	0.7452
12	(412)	Elisabetha	(891)	Gunhild	6450	0.0391 3015	1.1729
13	(311)	Claudia	(1397)	Umtata	6552	0.1430 1140	0.3208
14	(1251)	Hedera	(1492)	Oppolzer	7005	0.1158 8773	0.3951
15	(76)	Freia	(1692)	Subbotina	8880	0.1604 1612	0.2827
16	(1079)	Mimosa	(1100)	Arnica	9767	0.0558 9803	0.8068
17	(143)	Adria	(469)	Argentina	9895	0.0524 2515	0.8595
18	(1082)	Pirola	(1782)	1931 TL_1	11189	0.1036 0671	0.4309
19	(171)	Ophelia	(1581)	Abanderada	11195	0.0433 3501	1.0302
20	(1651)	1936 HD	(1856)	Ruzena	12541	0.0588 0106	0.7510

We shall define the „dynamical" duration of proximity as the time interval during which the gravitational effect of one asteroid upon the other attains an amount that is equal to, or larger than, some small quantity, settled *a priori*. In other words, this is a time interval delineated by the integration limits of the system of differential equations of the perturbed asteroid's osculatory elements, involved in the perturbation calculus.

It has been established in [4] that, concerning the quasi-complanar asteroids and the application of the perturbation calculus set forth therein, it is the value of the W-component of the perturbing acceleration which is decisive for the integration limits

$$W = U\,\rho^{-3}\zeta_i, \qquad U = k\,w\,m_i\sqrt{p}, \qquad \zeta_i = (\mathbf{R}\vec{\rho}). \tag{3}$$

Here w is the „integration step", m_i the assumed mass of the perturbing asteroid, p and $\mathbf{R}$, respectively, the orbit parameter and the unit vector of the perpendicular to the perturbed asteroid's orbital plane. Let the integration limits $-t_0$ and t_0 be symetrical with respect to t_p. These are chosen such that

$$W(t_0) = X, \tag{4}$$

where X is a small quantity given *a priori*. If we are dealing with a regular proximity, it follows from (1)

$$\rho^{-3} = (\rho_p^2 + V_p^2\tau^2)^{-3/2},$$

which gives, in view of (1), (3) and (4),

$$(A+B\tau_0)\,(C+D\tau_0^2)^{-3/2}=Y, \tag{5}$$

$$A=(\mathbf{R}\vec{\rho}_p), \quad B=(\mathbf{V}_p\,\mathbf{R}), \quad C=\rho_p^2, \quad D=V_p^2,$$

$$Y=X\,U^{-1}=X(k\,w\,m_i\,\sqrt{p})^{-1}.$$

The root τ_0 of Eq. (5) gives the dynamical semi-duration of proximity

$$\tau_0=k|t_0-t_p|.$$

Before the actual solution of the equation for τ_0, we shall write it in the following form:

$$\tau_0=[E(A+B\tau_0)^{2/3}-F]^{1/2}, \tag{6}$$

$$E=D^{-1}Y^{-2/3}=(UX)^{2/3}\;V_p^{-2}, \qquad F=C\,D^{-1}=(\rho_p/V_p)^2.$$

Its solution proceeds by successive approximations, taking $\tau_0=0.001$ or 0.002.

In our previous studies we took as an example a supposed proximity of asteroids (205) Martha and (992) Swasey, such as would occur around proximity points of their respective orbits. There the elements of proximity were somewhat different from those in Table I, as we had

$$\rho_p=0.000\,0379, \qquad V_p=0.057\,8599, \qquad m_i=10^{-13}, \qquad |t_0-t_p|=0^d.15.$$

By taking $X=0''.0001$, Eq. (6) becomes

$$\tau_0=[0.004\,6550\,(0.000\,0379-0.001\,6594\,\tau_0)^{2/3}-0.000\,0004]^{1/2}.$$

The series of the successive root values of τ_0 is

0.002, 0.002 131, 0.002 126, 0.002 126,

that is $\tau_0=0.1236$ mean days. This is in full agreement with the result in [4].

The dynamical duration of regular proximities can be determined in another way in proceeding from the ideas advanced in [3]. It has been shown there that

$$\int_{-t_0}^{t_0} W\,dt=\lambda_0\,W_P, \qquad W_P=2\,m_i\,\sqrt{p}\,(\vec{\rho}_p\,\mathbf{R})\rho_p^{-2}\,V_p^{-1},$$

$$\lambda_0=\tau_0(c^2+\tau_0^2)^{-1/2},\; c=\rho_p V_p^{-1},\; \tau_0=k|t_0-t_p|. \tag{7}$$

λ_0 tends to unity, if the time interval $2\tau_0$ of integration is increasing. For $\lambda_0=1$ we obtain W_P as the so called quasi-total perturbation effect. However, in view of the small or moderate amounts of the perturbations $\lambda_0 W_P$, it can be taken $\lambda_0=0.9$, or 0.99, etc. Then the dynamical duration of proximity obtained from Eq. (7) is

$$2\tau_0=2\,K\,c, \qquad K=\lambda_0(1-\lambda_0^2)^{-1/2}=\mathrm{tg}\;(\mathrm{arc\;sin}\;\lambda_0), \tag{8}$$

by which 0.9, or even 0.99, etc. of the „full" amount of the perturbation action is assured.

Table II summarizes the semi-values of the dynamical durations of proximity of asteroids, listed in Table I, for $\lambda_0=0.95$, 0.99 and 0.999.

A more accurate value of λ_0 is obtained with $\lambda_0=1-(\Delta/A)$, where A is some perturbation amount and $\Delta=A-\lambda_0 A$ an a priori given accuracy.

We found in [3], for the event of the asteroids (205) Martha and (992) Swasey proximity, that $W_p=+0''.0326$. For the factual calculus of the perturbation changes of the asteroid (992) Swasey's motion elements, under the perturbing action of the asteroid (205) Martha, the mass of which is assumed as $m_i=10^{-13}$, an accuracy $\Delta=0''.001$ is quite sufficient. Thus it will be

$$\lambda_0=0.969, \qquad K=3.922, \qquad \tau_0=0.136 \text{ mean days.}$$

Moreover, in consequence of our having obtained for the perturbation changes of the (992) Swasey's motion elements

$$\Delta\Omega=+0''.09, \qquad \Delta i=+0''.02,$$

these amounts will keep unchanged, with the same accuracy, if multiplied by $\lambda_0=0.95$. This yields $\tau_0=0.106$ mean days. All these results are in full harmony with inferences made in previous papers.

Table II

No.	Asteroids pair		c	$\lambda_0=0.95$	0.99	0.999
1	(215)	(1851)	0.00258	0.008	0.018	0.038
2	(227)	(1737)	0.00420	0.013	0.029	0.094
3	(389)	(972)	0.00313	0.010	0.022	0.070
4	(960)	(1818)	0.03988	0.121	0.280	0.891
5	(763)	(985)	0.00767	0.023	0.054	0.171
6	(703)	(1130)	0.00706	0.021	0.050	0.158
7	(1736)	(1759)	0.00941	0.029	0.066	0.210
8	(205)	(992)	0.03477	0.106	0.244	0.777
9	(938)	(1815)	0.01350	0.041	0.095	0.302
10	(954)	(1898)	0.02117	0.064	0.149	0.473
11	(110)	(1393)	0.05972	0.182	0.419	0.473
12	(412)	(891)	0.09582	0.292	0.672	2.141
13	(311)	(1397)	0.02662	0.081	0.187	0.595
14	(1251)	(1492)	0.03561	0.107	0.247	0.766
15	(76)	(1692)	0.03218	0.098	0.226	0.719
16	(1079)	(1100)	0.10160	0.309	0.713	2.270
17	(143)	(469)	0.10978	0.334	0.770	2.543
18	(1082)	(1782)	0.06278	0.191	0.441	1.403
19	(171)	(1581)	0.15024	0.457	1.054	3.357
20	(1651)	(1856)	0.12397	0.377	0.870	2.770

In [5] a semi-analytical method of calculus of perturbations at the asteroids proximities has been presented and has shown that it is convergent provided the semi-time interval $\tau_0<c=\rho_p/V_p$. Now, it can be stated that

this requires, according to Eq. (8), $K<1$, which results in $\lambda_0<\sqrt{2}/2=0.707$. However, such a „low accuracy“ is acceptable only in case that the perturbing effects are very small.

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