Publ. Obs. Astron. Belgrade No. 50 (1995), 103 – 104

THE INTENSITY PROFILE IN THE MODEL OF "FRACTIONAL" OSCILLATOR

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In Nigmatullin (1992) it has been shown that partial summation of the temporal "Cantor fingers" convoluted with a smooth function $f(t)$ can be substituted in the limit $N \to \infty$ by temporal fractional integral (TFI) taken from the smooth function. The importance of this general approach to physical understanding of TFI has been stressed recently in the review Olemsky and Flat (1993). The main result of Nigmatullin (1992) can be expressed in the form

$$< J(t) >= \lim_{N\to\infty} J_N(t) = \lim_{N\to\infty} \frac{1}{2^N} \sum_{m=1}^{2^N} f(t - t_m^{(N)}) =$$

$$= B(\nu)T^{-\nu} \frac{1}{\Gamma(\nu)} \int_0^t (t-\tau)^{\nu-1} f(\tau) d\tau = B(\nu)T^{-\nu} D^{-\nu}[f(t)] \qquad (1)$$

For $0 < t/T < 1$, $\nu = \ln 2/\ln(1/\xi)$, $0 < \xi < 1/2$. Here $t_m^{(N)}$ is the N-th generation of the points of the Cantor set, [0,T] is an initial interval of the location of the Cantor set, ξ is an initial length of the Cantor column, $\nu = \ln n2/\ln(1/\xi)$ is the fractal dimension of the Cantor set,

$$B(\nu) = (1-\xi)^{-\nu} 2^{-\frac{1+\nu}{2}}$$

is the constant, depending on ν, $D^{-\nu}[f(t)]$ is the symbol of TFI determinated from (1). This result can be applied to consideration of selfsimilar collisions which can be generated by random collision force.

Based on this relationship (1) it is easy to outline the new approaches to consideration of collision motion.

Let us suppose that the process of interaction of the physical system considered with external surroundings is described by means of random and collisional force. In this case the Newton's equation assums the form.

$$\frac{m}{T^2} \frac{d^{1+\nu}(\Delta \bar{r})}{du^{1+\nu}} = B(\nu) \bar{F}(\bar{r}, \bar{v}, t), \quad 0 < \nu < 1, \ 0 < u = t/T < 1 \qquad (2)$$

The equation (2) can be used for description of the collision and brownian processes where irreversibility due to interactions (the processes of "remnant" memory (Nigmatullin, 1992; Olemsky and Flat, 1993)), is taken into account "automatically". If the force $\overline{F}(\overline{r},\overline{v})$ coincides with additive sum of elastic and friction force (for simplicity we consider onedimensional case)

$$\overline{F}(\overline{r},\overline{v}) = -\gamma\Delta\overline{v} - k\Delta x \tag{3}$$

then equation (2) is reduced to the equation of "fractional" oscillator with losses describing the movement of a system in a medium with selfsimilar collisions

$$\frac{\mathrm{d}^{1+\nu}x}{\mathrm{d}u^{1+\nu}} + B(\nu)(2\lambda T\frac{\mathrm{d}x}{\mathrm{d}u} + \omega_0^2T^2x) = 0, \qquad 0 < \nu < 1, \; u = t/T \tag{4}$$

Irreversibility and collisions are taken into account by the exponent ν. At $\nu = 0$. we obtain solution for amplitude which leads to the Lorentz profile for intensity. At $\lambda = 0$ it is possibble to obtain the solution for $x(t)$ in the form

$$\frac{x(t)}{x(0)} = E_{1+\nu}(-\tau^{1+\nu}) \tag{5}$$

where $\tau = \omega_p t$, $\omega_p = [B(\nu)(\omega_0 T)^2]^{1/(1+\nu)}/T$ is the natural frequency of "fractional" oscillator, $E_\alpha(z) = \sum_{n=0}^{\infty}(z^n/(\Gamma(\alpha n+1)))$ is Mittag-Lefleur function.

The Fourier transformation of $x(t)$ gives the amplitude $a(\omega)$, which leads to the profile of intensity $P(\omega) = a^*(\omega)a(\omega)$.

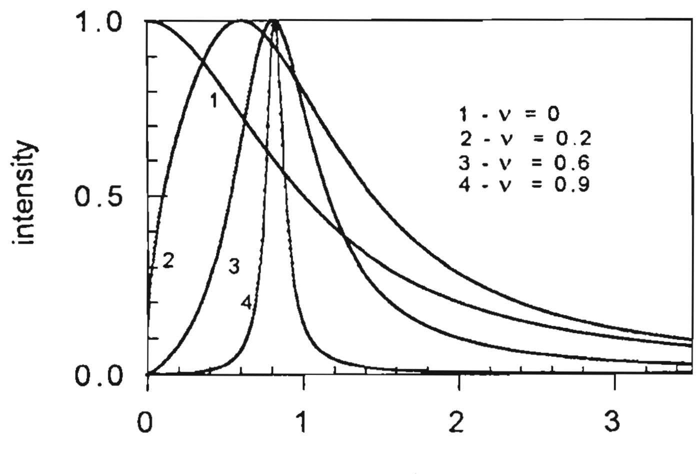

Fig. 1.

In the Figure the distribution of intensity vs frequency ω in arbitrary untis for various ν is shown.

Thus the conception of TFI and its generalizations can find wide application for description of dynamical processes in physics having fractal grounds in their origin. This model of fractional oscillator can serve as the ground for consideration of line broadening in plasma.

References

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