LOGIC COLLOQUIUM 87

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Título: Bases de Gröbner y Deducción

Resumen

En este trabajo estudiamos aplicaciones de las bases de Gröbner de ideales de $Z/Z2(X_1,\ldots,X_n)$ para la resolución efectiva de problemas del Cálculo Proposicional.

Comenzamos resumiendo algunos conceptos y resultados sobre los sistemas de reescritura. A continuación, estudiamos las bases de Gröbner en $Z/Z2(X_1,\ldots,X_n)$ y algoritmos que nos permiten resolver el problema de pertenencia a un ideal. Definimos explícitamente un isomorfismo Θ entre el anillo booleano correspondiente al álgebra de Lindenbaum-Tarski de las proposiciones y el anillo cociente

$$Z/Z2(X_1,\ldots,X_n) \,/\, (X_1^2+X_1,\ldots,X_n^2+X_n) ,$$

demostrando que una proposición Q es una consecuencia de $P_1,\ldots,P_m$ si y sólo si $\Theta(Q)+1$ pertenece al ideal generado por $\Theta(P_1)+1,\ldots,$ $\Theta(P_m)+1$, y reduciendo posteriormente este problema al caso de los polinomios.

Una vez reducido el problema de la deducción al de la pertenencia a un ideal del anillo de polinomios, exponemos en la última parte del trabajo, algoritmos para decidir si una proposición es consecuencia de un conjunto dado, si un conjunto de axiomas es consistente, si varios conjuntos de axiomas son equivalentes, y otros problemas del Cálculo Proposicional.

Este enfoque presenta varias ventajas frente a los métodos usuales de prueba automática: no es refutacional, su utilidad no es sólo la prueba de teoremas, y, sobre todo, es el único que permite introducir la "compilación" de axiomas, con lo que el trabajo de la deducción se simplifica enormemente.

key words: Theorem proving, Gröbner basis, Propositional Calculus.

The method of hypersequents in many-valued logic

by
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Hypersequents are generalization of Gentzen sequents in which one is allowed to deal with finite sequences (or sets) of ordinary sequents. The use of hyper- sequents enable us to provide cut-free formulations of some known multiple- valued logics. (This extends results of [1] and [2]).

Definition: Let L be a language. A hypersequent is a creature of the form:

$$(\Gamma_1 \Rightarrow \Delta_1), (\Gamma_2 \Rightarrow \Delta_2), ..., (\Gamma_n \Rightarrow \Delta_n)$$

where Γ_i, Δ_i are finite sequences of formulas of L and $n \geq 0$.
We shall use G, H as metavariables for hypersequents.

We shall use the usual stock of rules, but with side-sequents allowed. However, we have two kinds of structural rules: The external ones, which work on a hypersequent, and the internal ones, which change the structure of a component of a hypersequent.

L_3 **(the three-valued logic of Lukasiewicz)**: This is obtained by taking all the usual axioms and rules, except for the two internal contractions. In addition it has the following structural rule:

$$\frac{G, (\Gamma_1, \Gamma_2, \Gamma_3 \Rightarrow \Delta_1, \Delta_2, \Delta_3) \qquad G', (\Gamma_1', \Gamma_2', \Gamma_3' \Rightarrow \Delta_1', \Delta_2', \Delta_3')}{G, G', (\Gamma_1, \Gamma_1' \Rightarrow \Delta_1, \Delta_1'), (\Gamma_2, \Gamma_2' \Rightarrow \Delta_2, \Delta_2'), (\Gamma_3, \Gamma_3' \Rightarrow \Delta_3, \Delta_3')}$$

LC **(Dummet's super-intuitionistic logic)**: This has all the usual rules, with the usual intuitionistic limitation to at most one formula to the right of $\Rightarrow$, and in addition the following structural rule:

$$\frac{G_1, (\Gamma_1', \Gamma_1 \Rightarrow \Delta_1) \qquad G_2, (\Gamma_2', \Gamma_2 \Rightarrow \Delta_2)}{G_1, G_2, (\Gamma_2', \Gamma_1 \Rightarrow \Delta_1), (\Gamma_1', \Gamma_2 \Rightarrow \Delta_2)}$$

LC_3 **(The three-valued extension of** LC**)**: Like L_3, but with the above intuitionistic limitation and with the internal contractions.

RM_3 **(The 3-valued extension of RM)**: All the usual rules **except for the two weakening rules.** In addition it has the follwing two structural rules:

$$\frac{G_1, (\Gamma_1 \Rightarrow \Delta_1) \qquad G_2, (\Gamma_2 \Rightarrow \Delta_2)}{G_1, G_2, (\Gamma_1, \Gamma_2 \Rightarrow \Delta_1, \Delta_2)}$$

$$\frac{G, (\Gamma_1, \Gamma_2 \Rightarrow \Delta_1, \Delta_2)}{G, (\Gamma_1 \Rightarrow \Delta_1), (\Gamma_2, \Gamma' \Rightarrow \Delta_2, \Delta')}$$

References

[1] G. Pottinger "Uniform,cut-free formulations of T,S4 and S5",(abstract), J.S.L.,vol. 48 (1983),p. 900.

[2] A. Avron "A constructive analysis of RM" (to appear)

Diverse Classes

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For any μ and any class K of structures, let $\mathbf{IE}(\mu, K)$ denote the supremum of the cardinalities of antichains in the partial ordering of members of K with cardinality μ induced by elementary imbedding. We introduce the notion of a diverse class and show

Theorem. *For any diverse class K, $\mathbf{IE}(\mu, K) \geq min(2^\mu, \beth_2)$.*

The notion of a diverse class codifies the properties needed to show the following result of Shelah.

Corollary. *Let T be a superstable theory. If $|T| < 2^\omega$ and $|D(T)| = |T|$ then $\mathbf{IE}(\mu, T) \geq min(2^\mu, \beth_2)$.*

MODAL SEMANTICS FOR SUPERVALUATIONAL FREE LOGIC (Abstract)

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Improving Van Fraassen's ideas, E. Bencivenga developed a supervaluational semantics for free logic. He considered a certain set of structures instead of a single one. It is well known how Kripke's relational semantics has been accommodated to non-standard first order logics, such as intuitionistic logic or many valued-logic. In a similar way, we can define a certain kind of relational models for free logic, and prove that the set of supervaluationally valid sentences is characterized by the class of all such models, and by some subclasses of these models. We also show how to translate free logic sentences into a modal language and finally consider the modal logic determined by our models.

Please disregard the former abstract. Open Mapping. . . .

ABSTRACT

p-ADICALLY CLOSED DOMAINS. L. Bélair. Let p be a prime, $\mathbb{Q}_p$ the field of p-adic numbers, and $\mathcal{C}(\mathbb{Q}_p)$ the ring of continuous functions $\mathbb{Q}_p \to \mathbb{Q}_p$ which are definable. We study the elementary theory of the quotient rings $A(\mathcal{P}) = \mathcal{C}(\mathbb{Q}_p)/\mathcal{P}$, where $\mathcal{P}$ is a non-maximal prime ideal, by realizing $\mathcal{P}$ as the kernel of an evaluation map at some nonstandard point.

<u>Proposition</u>. For all $\mathcal{P}$, $A(\mathcal{P})$ is a henselian valuation ring whose residue field is a p-adically closed field, and whose value group is divisible. Hence, all such are elementarily equivalent. Moreover the field of fractions of $A(\mathcal{P})$ is p-adically closed.

We get a natural axiomatization pCD of this theory as a completion of the theory pCLR in [Bé] which axiomatizes henselian local rings with p-adically closed residue field. It follows that pCD is decidable; furthermore it has E.Q. in a natural language. Models of pCD can be characterized as the valuation rings of p-adically closed fields corresponding to convex subgroups of the value group of the p-adic valuation. These "p-adically closed domains" are the p-adic ánalog of the real closed rings of Cherlin-Dickmann [CD].

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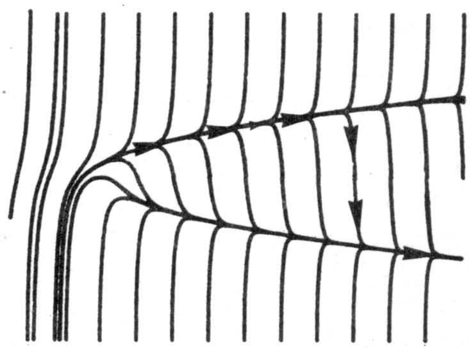
[CD] G. Cherlin and M. Dickmann. Real Closed Rings II, ANN. PURE APPL. LOGIC 25 (1983), 213-231.

Descriptive set theory and nonstandard asymptotics

The external sets of nonstandard analysis play an important part in handling asymptotic problems. Nonstandard asymptotics has notably been successful in relation to singular perturbations of differential equations and pointwise approximation by divergent asymptotic series.

We first consider the external convex subsets of the nonstandard real line. Not unlike intervals, they can be classified in four definite types, with boundaries consisting of translations of convex external additive groups. In asymptotic analysis, the external convex sets notably act as orders of magnitude, and as domains of validity of asymptotic approximations and reasonings; one can say that an asymptotic phenomenon is localized, once the boundary sets are known.

Secondly we consider the problem of getting a global description of a function, in case local, changing, asymptotic behaviour is known. The nonstandard approach to this problem, which classically is known as matching, uses general permanence principles, originating from the incompatibility of certain classes of external sets. For example, no external set can be defined both by a formula of the form $(\exists^{st}x)(\phi(x,y))$ or of the form $(\forall^{st}x)(\psi(x,y))$, where ϕ and ψ are classical.



Example of changing asymptotic behaviour: Solutions of the differential equation $dY/dX = Y^2-X$.

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Une formulation élémentaire du principe de récursion

par Michel BONNARD (Université de Perpignan)

Résumé :

Quand un mathématicien (non logicien) utilise la récursion (transfinie) c'est généralement pour fabriquer de proche en proche, dans un ensemble ordonné, des éléments de plus en plus grands, jusqu'à en obtenir un qui soit maximal parmi ceux à qui il s'intéresse. Ce genre de choses peut se faire sans parler d'ordinaux (ni même d'ensembles bien ordonnés) en utilisant l'énoncé suivant :

Soit X un ensemble (partiellement) ordonné dans lequel existe un "sélecteur de majorants". Alors tout élément de X appartient à une chaîne n'ayant pas de majorant strict.

L'axiome du choix, si on l'admet, dit que dans tout ensemble ordonné il y a un sélecteur de majorant (qui opère par choix arbitraire). On retombe ainsi sur un lemme (méconnu) de Tukey (d'où le lemme de Zorn et ses conséquences).

Par contre, si on n'admet pas l'axiome du choix, il peut exister des sélecteurs de majorants qui n'opèrent pas par choix arbitraire. Par exemple notre énoncé peut servir à démontrer que toute mesure positive sur un clan se prolonge au σ-clan engendré (la démonstration, par récursion, de ce théorème est (plus ou moins) connue, mais ce n'est pas celle qui figure habituellement dans les livres car les non-logiciens sont rebutés par le formalisme des ordinaux).

Multi - Valued n-Sequential First Order Predicate Logic

Piotr Borowik

Abstract

In the sequent method of construction of multi-valued first order predicate calculus the main concept is a sequent .The sequent is defined as a pair of finite sets of formulae.It is possible that the sequent in the n-valued predicate calculus can be defined as ordered n-tuple finite sets of formulae ,but with another interpretation.If $X_1, X_2, \ldots, X_n$ are finite (possibly empty) sets of formulae then a sequent is an ordered n-tuple $X_1 \vdash X_2 \vdash \ldots \vdash X_n$ .By an overfilled sequent we mean a sequent $X_1 \vdash X_2 \vdash \ldots \vdash X_n$ such that $X_j \cap X_k \neq \emptyset$ for some $j \neq k$. This first order predicate calculus consists of rules applied to either overfilled sequents or sequents obtained earlier and is of an algorithmic character.

The completeness theorem holds for a calculus constructed in this way.

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Model theory of elementary pairs of models

Elisabeth BOUSCAREN

If T is a complete first-order theory in a language L, let $L^* = L \cup \{\mathcal{M}\}$, where $\mathcal{M}$ is a new unary predicate symbol.

We consider elementary pairs of models of T, $M \prec N$, $M \neq N$, as L^* structures, with M interpreting the predicate $\mathcal{M}$. It is clear that they form an elementary class which is not in general complete.

The main idea behind the study of pairs in this extended language is to see to what extent it is possible to express properties of the theory T in a simpler way with the help of this new language.

We recall previous results which show that in many cases it turns out to be the case (finite cover property [B.Poizat], Dimensional Order Property [E.Bouscaren]), and present some more recent results and natural questions on the expressive power of the model theory of pairs of models in this new language (joint work with B.Poizat).

[E.Bouscaren] 1- Thèse de Doctorat d'état, Université Paris 7, 1985

2- Dimensional Order Property and pairs of models, preprint.

[E.Bouscaren et B.Poizat] Des belles paires aux beaux uples, to appear in the Journal of Symbolic Logic.

[B.Poizat] Paires de structures stables, Journal of Symbolic Logic 48 (1983) , 239-249.

CHRIS BRINK, *A verisimilar ordering of theories phrased in a propositional language.* *

Popper introduced the concept of verisimilitude in the early sixties; Miller and Tichy deflated his definition in the early seventies. These facts, and the ensuing debate on verisimilitude, are well chronicled in Graham Oddie's book Likeness to Truth (Reidel, 1986). The present contribution is peripheral to that context, being the serendipitous application to verisimilitude of a certain algebraic construction. Namely, to any structure there corresponds its *power structure*, essentially built up by taking the *power relation* of each relation in that structure. In particular we can associate with any ordering its *power ordering*. It turns out that propositional formulae, when viewed as sets of sets of propositional variables, are naturally if unconventionally ordered by a power ordering. This ordering is presented as a candidate verisimilar ordering of theories phrased in a propositional language. It obeys the following principles of verisimilitude.

(1) No false theory has greater verisimilitude than any true theory.

(2) For true theories conjunction increases verisimilitude and disjunction decreases verisimilitude.

(3) The verisimilitude of a true theory is not increased by conjoining to it a false theory. (It is either decreased or a non-comparable theory results.)

(4) The verisimilitude of a false theory may be increased or decreased or neither by conjoining to it a true theory.

(5) The verisimilitude of true theory is decreased by disjoing to it a false theory.

(6) The verisimilitude of a false theory is not decreased by disjoining to it a true theory. (It is either increased or a non-comparable theory results.)

(7) For false theories verisimilitude may be increased or decreased or neither by conjunction as well as by disjunction.

* (Joint work with J. Heidema)

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PERMUTATION MODELS AS REPRESENTATIONS OF ZERO-DIMENSIONAL GROUPS

The topology of the automorphism group of a permutation model PM is related to the variants of the axiom of choice which are valid in PM. For instance, PM satisfies global multiple choice, iff its automorphism group is locally compact (Brunner, Rubin 1986). In this talk some theorems of this kind are surveyed and a general persistency result is given.

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COMPACT MODELS

Every model $\mathfrak{M}$ has an associated logic, $\mathcal{L}_{\mathfrak{M}}$, wich is related to $\mathfrak{M}$ as ω-logic is related to the model $(\omega,<)$. Given a structure $\mathfrak{A}$ whose similarity type has a unary predicate U and includes the similarity type $\tau_{\mathfrak{M}}$ of $\mathfrak{M}$, we say that $\mathfrak{A}$ is an $\mathfrak{M}$-model iff $U^{\mathfrak{A}}$ is $\tau_{\mathfrak{M}}$-closed and $(\mathfrak{A}\restriction\tau_{\mathfrak{M}})|U^{\mathfrak{A}} \cong \mathfrak{M}$. In other words, an $\mathfrak{M}$-model is a model with a relativized reduct isomorfic to $\mathfrak{M}$.

It is clear that if $\mathfrak{M}$ is an infinite model, the logic $\mathcal{L}_{\mathfrak{M}}$ is at most $\|\mathfrak{M}\|$-compact, that is, compact for sets of $\leq\|\mathfrak{M}\|$ sentences.Thus, we say that $\mathfrak{M}$ is compact iff $\mathcal{L}_{\mathfrak{M}}$ is $\|\mathfrak{M}\|$-compact. If $\tau_{\mathfrak{M}}$ and $\mathfrak{M}$ are countable, then $\mathfrak{M}$ is compact iff $\mathfrak{M}$ is saturated (cf. [1]). We show that every saturated and every special model $\mathfrak{M}$ whose similarity type is of power $\leq\|\mathfrak{M}\|$ is compact,but that there are uncountable compact models wich are neither saturated nor special.

References

[1] M.MORLEY <u>Countable models with standard part</u>, Proceedings of the Fourth International Congress for Logic, Methodology and Philosophy of Science, Bucharest 1971.

ON DUMMETT's LC QUANTIFIED

GIOVANNA CORSI

Let Q-LC be the logic that results from adding Dummett' s axiom $(A \to B \lor B \to A)$ to intuitionistic predicate logic. We give an axiomatization of Q-LC in a sequent calculus G-LC and we prove the cut elimination theorem for it.

A sequent $M \twoheadrightarrow N$ is an ordered pair of finite sets (possibly empty) of formulas and the notation $M, A \twoheadrightarrow N$ is an abbreviation of $M \cup \{A\} \twoheadrightarrow N$. Analogously for $M \twoheadrightarrow A, N$.

Axioms of G-LC : $M, A \twoheadrightarrow A, N$; $M, \int \twoheadrightarrow N$.

Rules of G-LC : G-LC contains among its rules the following one which allows the simultaneous introduction of universally quantified and implicative formulas in the consequent of a sequent,

$$\frac{\begin{array}{l} M_1, A_1 \twoheadrightarrow B_1, A_2 \to B_2, \ldots, A_n \to B_n, \forall x D_1(x), \ldots, \forall x D_m(x) \\ \vdots \\ M_2, A_n \twoheadrightarrow A_1 \to B_1, \ldots, A_{n-1} \to B_{n-1}, B_n, \forall x D_1(x), \ldots, \forall x D_m(x)) \\ M_{n+1} \twoheadrightarrow A_1 \to B_1, \ldots, A_n \to B_n, D_1(w_1), \forall x D_2(x)), \ldots, \forall x D_m(x)) \\ \vdots \\ M_{n+m} \twoheadrightarrow A_1 \to B_1, \ldots, A_n \to B_n, \forall x D_1(x), \forall x D_2(x), \ldots, D_m(w_m) \end{array}}{\bigcup_{i=1}^{n+m} M_i \twoheadrightarrow A_1 \to B_1, \ldots, A_n \to B_n, \forall x D_1(x), \ldots, \forall x D_m(x))} L_{n,m}$$

where $w_1,\ldots,w_m$ are all different variables and none of them occurs in the conclusion of the rule and n, $m \geq 0$.

$w_1,\ldots,w_m$ are said to be the special variables of $L_{n,m}$.

The other rules of G-LC are the usual ones for predicate intuitionistic logic.

Let G-LC* be the calculus G-LC plus the following cut rule :

$$\frac{M \twoheadrightarrow A, N \qquad A, P \twoheadrightarrow Q}{M, P \twoheadrightarrow N, Q} \text{cut}$$

The formula A is said to be the <u>cut formula</u> .

LEMMA 2.3 Let Ω be a proof of the sequent $M \twoheadrightarrow N$ in G-LC*. Then there is a proof Ω^* of $M \twoheadrightarrow N$ in G-LC.

LEMMA 2.4 A sequent $M \twoheadrightarrow N$ is provable in G-LC iff Q-LC $\vdash \bigwedge M \to \bigvee N$ where $M, N \neq \emptyset$.
$\twoheadrightarrow N$ is provable in G-LC iff Q-LC $\vdash \bigvee N$, where $N \neq \emptyset$.
$M \twoheadrightarrow$ is provable in G-LC iff Q-LC $\vdash \bigwedge M \to \int$, where $M \neq \emptyset$.

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RESUMEN DE LA COMUNICACION.-

La presente propuesta se estructura en dos planos que se corresponden con los dos planos básicos del análisis de los lenguajes en general: la lógica y la gramática.

La propuesta consiste en utilizar lenguajes de primer orden para la caracterización de las lógicas de la creencia de fragmentos relevantes de oraciones del castellano y al mismo tiempo, y sin que por ello se resienta la coherencia o la elegancia del análisis en su conjunto, usar lenguajes de orden superior (I.L. y M.L.p) para la caracterización de la estructura gramatical, tanto sintáctica como semántica, de ese mismo fragmento. En concreto:

i) Nivel lógico.- Propuesta de construir análogos de los sistemas clásicos de lógica modal para definir operadores epistémicos de creencia. Presentación de un sistema con su correspondiente prueba de completud.

ii) Nivel gramatical.- Propuesta de uso del operador Dthat de Kaplan, reformulado por Klein, para formalizar en I.L. las oraciones de creencia. Tratando las clausulas introducidas por "que" dentro de una teoría de las expresiones referenciales.

Esta comunicación es parte del conjunto de investigaciones que estoy completando para la redacción de mi tesis doctoral dirigida por el profesor Hierro Sanchez-Pescador.

R. Crespo - 2

Les envío un breve resumen de la comunicación que desearía presentar en el *Logic Coloquium 87-Granada* por si consideran posible su aceptación. Mis datos son:

Rafael Crespo Arce, Calle Villa de Marín 43, Madrid 28029. Telefono 201 04 18.

También pueden comunicarme su decisión al Departamento de Lógica de la Universidad Autónoma de Madrid donde realizo mis investigaciones bajo la dirección del profesor Hierro Sanchez-Pescador.

Adjunto a este resumen la fotocopia del justificante de la transferencia hecha a la cuenta por ustedes indicada. En caso de no ser aceptada la comunicación desearía igualmente ser inscrito para asistir a las sesiones del coloquio.

En caso de ser aceptada indiquenme si debo presentarla en Inglés o en Castellano.

Atentamente

R. Crespo

Abstract of "TOPOLOGICAL DUALITY OF DIAGONALIZABLE ALGEBRAS"

PAOLA D'AQUINO

The concept of Diagonalizable Algebra (DA) has been introduced by Magari in order to provide an algebraic treatment of incompleteness logical phenomena. (We recall that the theory of DA's can be translated into the modal logic GL.) A first representation theorem for DA's has been obtained as a particular case of Halmos' duality for hemimorphisms. Our aim is to provide a purely topological representation of DA's by introducing bitopological spaces, i.e. Stone spaces enriched with a second topology.

In this direction the first step dates back to 1974: before DA's had been introduced, Simmons studied properties of theories from a topological point of view. E.g. Gödel's theorems are equivalent to the existence of isolated points and Löb's property is translated into the concept of scattered space. In 1981 Esakia gave a topological representation of a particular class of DA's by interpreting the operation σ as a derived set operator d w. r. to a suitable topology: $\langle \wp(X), d \rangle$ is a DA iff X is a scattered space. This idea was developed by Buszkowski and Prucnal in 1984.

Working on these ideas, we achieve a purely topological representation of all DA's. Besides, we obtain a topological characterization of the continuous maps corresponding to DA's morphisms. Thus the Stone duality is extented to DA's.

Also, bitopological spaces allow us to translate algebraic concepts as τ-filter, quotient algebra, subalgebra, direct product. Some applications are proposed.

THE PSEUDO MEET-ACCESSIBLE NONBRANCHING DEGREE

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Abstract

Definition 1 Let $\mathbf{A}$ be a set of r.e. degrees. The closure $CL(\mathbf{A})$ of A is defined to be the least set $\mathbf{B}$ such that

(1) $\mathbf{A} \subseteq \mathbf{B}$

(2) $\forall n \geq 0 \forall \mathbf{a}_0, \ldots, \mathbf{a}_n \in \mathbf{B}(\mathbf{a}_0 \cap \cdots \cap \mathbf{a}_n \text{ exists} \rightarrow \mathbf{a}_0 \cap \cdots \cap \mathbf{a}_n \in \mathbf{B})$

(3) $\forall n \geq 0 \forall \mathbf{a}_0, \ldots, \mathbf{a}_n \in \mathbf{B}(\mathbf{a}_0 \cup \cdots \cup \mathbf{a}_n \in \mathbf{B})$

If we omit clause (2), then $\mathbf{B}$ is called the closure of $\mathbf{A}$ under joins and is denoted by $CL_j(\mathbf{A})$.

Definition 2 An r.e. degree $\mathbf{a}$ is called pseudo meet-accessible if there is a set $\mathbf{A}$ of r.e. degrees such that $\mathbf{a} \in CL(\mathbf{A})$ and $\mathbf{a} \notin CL_j(\mathbf{A})$.

In this paper we prove that there is a nonbranching degree which is pseudo meet-accessible.

In this paper there are three main theorems:

Th.1 There are r.e. degrees $\mathbf{a}, \mathbf{b}_0, \mathbf{b}_1, \mathbf{c}$ such that $\mathbf{c}$ is low, $\mathbf{0} <_T \mathbf{c} <_T \mathbf{a}$, $\mathbf{b}_0 \cap \mathbf{b}_1 = \mathbf{c}$ and $\mathbf{b}_0 \not\leq_T \mathbf{a}$, $\mathbf{b}_1 \not\leq_T \mathbf{a}$.

Th.2 Let $\mathbf{a}$, $\mathbf{c}$ be r.e. degrees, $\mathbf{c} <_T \mathbf{a}$ and $\mathbf{c}$ is low, then there are $\mathbf{e}, \mathbf{d}$ such that $\mathbf{e}$ is nonbranching degree, $\mathbf{c} \leq_T \mathbf{e} <_T \mathbf{a}$, $\mathbf{d} |_T \mathbf{c}$ and $\mathbf{c} \cup \mathbf{d} = \mathbf{e}$.

Th.3 There is a psedo meet-accessible nonbranching degree.

A CONSERVATION RESULT OVER POLYNOMIAL TIME ARITHMETIC

In (1) Buss defines a weak system of arithmetic S^1_2 whose provably recursive functions (having appropriate graphs) are exactly those functions computable in deterministic polynomial time. We reformulate Buss's system by introducing a language and certain axioms which are meant to characterize, not the usual model ω, but the tree $2^{<\omega}$. In this language, the so called PIND axioms take the very natural form $A(\phi) \wedge \forall x (A(x) \rightarrow A(x0) \wedge A(x1)) \rightarrow \forall x A(x)$, where A is a Σ^b_1 formula and ϕ denotes the empty word.

Definition The second order theory $pWKL_0$ consists of a set of basic axioms, the scheme of PIND induction for Σ^b_1 formulas, together with,

(a) the comprehension scheme $\forall x(A(x) \leftrightarrow B(x)) \rightarrow \exists X \forall x (x \in X \leftrightarrow A(x))$, where $A(x) := \exists y C(x,y)$ and $B(x) := \forall y D(x,y)$, C and D sharply bounded formulas.

(b) weak König's lemma, which is the axiom,

$\forall T$('T is an infinite sub-tree of $2^{<\omega}$' $\rightarrow \exists X$('X is a path through T')).

Note that the ω-models of $pWKL_0$ are the Scott systems, which always contain the recursive sets. However, despite the apparent strength of (a) and (b) this theory does not have more provably recursive functions than S^1_2. In fact:

Theorem $pWKL_0$ is conservative over S^1_2 with respect to Π^0_2 sentences.

(1) S.R.Buss, Bounded Arithmetic, Ph.D. Dissertation, Princeton University, 1985.

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Título: A first approach to abstract modal logics

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Abstract: We present a studyof four systems of modal logic (IM4,IM5,S4 and S5) within the theory of Abstract Logics of Brown,Bloom and Suszko,and the approach is similar to that Brown and Bloom for classical logic.We include an abstract definition of these logics and several characterization theorems by means of projective generation,offering an homogeneous presentation of these four systems.The key property common to all of them is the equality $C_{0}=C\circ I$,where C_{0} is the modal consequence operator using the Strong Rule of Necessitation,and C the same but with this rule applied only to theorems.The other key property is "Reductio ad Absurdum" which stated for several operators allows us to obtain S4 from IM4,IM5 from IM4, and S5 from either S4 or IM5.It plays a role similar to that played by transitivity in "double-Kripke-models" of the semantic studies of Dosen,Sotirov and others, and also to that played by the acceptance of multiple-conclusion sequents in the higher-level sequent-systems presented by Dosen.

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Is Cantorian Set Theory an iterative conception of set?

Scholars as important as G. Boolos, Ch. Parsons and H. Wang have thought it is plausible to understand as an iterative conception the Cantor's notion of set underlaying his well know work in two parts Beiträge zur Begründung der transfiniten Mengenlehre (1895-97). Such an interpretation means that the theory stated in Beiträge is consistent and that the contradictions discovered in it -- e.g., Russell's Paradox or that of the greatest ordinal or cardinal -- are actually the result of a misunderstanding of Cantor's theses. H. Wang, for instance, has claimed in favor of this affirmation.

However, in our opinion the theory of Beiträge is contadictory and belongs to the naïve conception of set. On the one hand, we cannot speak a single Cantor's set theory, since there are clearly two different theories in his writtings. The first one appears in all of the Cantor's published works about set theory, outstandingly Beiträge and Grundlagen (1883). This theory, the same in both cases, is a naïve one, with an irrestrict conception of what may be taken for a set. The second theory is found in Cantor's correspondence with Dedekind in 1899 and Jourdain from 1901 to 1906. This theory establishes a device for limiting the size of the sets and distinguish between collections which are sets and collections which are not sets. Such device must be considered, in our opinion, as a modificaton of his old theory in order to meke it consistent. The reasons by which we do not consider the theory of Beiträge is iterative, not even in Cantor's purposes, are the following:

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1. There are sources enough for stating without any reasonable doubt that Cantor himself was the first who discovered contradictions in his former theory, about at 1996. Thus to affirm that those contradictions were due to a misunderstanding of Cantor's writtings loses most of it likelihood.

2. If the theory of Beiträge would belong to the iterative conception, it should be consistent, and then it should not be necessary to work out a new theory. In fact, a more detailed reformulation of the former would has been enough and, besides, it cannot be explained why Cantor proposed a way for limiting the size of set in his later theory, a solution to the paradoxes less natural and more difficult to define than the idea underlaying the iterative conception.

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ON THE ROLE OF SELECTION OPERATORS IN GENERAL RECURSION THEORY

Among other reasons, the inclusion of selection operators in formulating a generalized recursion theory is desirable on the grounds that such theory is a reasonable generalization of the notion of a semirecursive relation. Often cited in support of this perception is the demonstration that the following two properties of semirecursive relations hold in the theory:

(*) A relation R is recursive $\Leftrightarrow$ R and $\neg R$ are both semirecursive.

(**) A function f is partial recursive $\Leftrightarrow$ f is a semirecursive relation.

When presented with such a situation, it is of interest to have examples showing that the inclusion of selection operators is necessary so that the above two properties hold in the generalized theory. In the first part of this paper we obtain such an example by exhibiting a model of a generalized recursion theory for which both (*) and (**) fail. The theory is formulated along the lines of Fenstad [1] and we give results that are pertinent to the construction of the model. The model is based on the notion of pseudo-reducibility, which is new. We define this by first generalizing the ordinary notion of enumeration reducibility for functions so that a function may be "enumeration reducible" to any finite list of functions. We then obtain pseudo-reducibility by placing certain restrictions on the procedures that carry out generalized enumeration reducibility. The net effect is that if a function f is pseudo-reducible to the functions $g_1,\ldots,g_n$, then for each $i=1,\ldots,n$, f can be extended to a function which is enumeration reducible (in the ordinary sense) to g_i. Thus, if f is total, then f is enumeration reducible to g_i for every $i=1,\ldots,n$. We define the desired model M as the collection of all functions which are pseudo-reducible to the semicharacteristic functions of a set S and its compliment, where S is recursively enumerable but not recursive. Property (*) fails in M (in the direction $\Leftarrow$) because S and $\bar{S}$ are both semirecursive in M, while S (and also $\bar{S}$) is not recursive in M (for every total function in M is recursive). We show that property (**) also fails in M (in the direction $\Leftarrow$) as a consequence of the failure of (*) and the fact that the semirecursive relations in M are closed under disjunction.

In the second part of this paper we use the method in the above outlined construction to throw some light on the connection between the selection and the monotonic operators. In particular, we show how to obtain models in which any number of monotonic operators, whose range is singleton, are forced in these models, while the selection operators are being kept out of the same models.

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Completely finite models for number theory

We start from the assertion that there is an upper bound B to the length (in bits) of any communication that can be made between mathematicians (or their computers). Hence extremely large numbers and relations between them can only be incompletely specified. Let L be a first-order language for number theory; a complete k-type is a maximal set of atomic formulae in $x_1,\dots,x_k$ all of length less than B, which is satisfiable in the standard model. A completely finite structure U for L is specified by: (1) bounds B for the length of formulae and K ($<$B) for the number of variables; (2) for each $k < K$ a subset E_k of the complete k-types. A truth-functional notion of satisfaction $U \models \psi(\Gamma)$ where $\psi = \psi(x_1,\dots,x_k)$ and $\Gamma \in E_k$ is defined by: (a) for ψ atomic $U \models \psi(\Gamma)$ iff $\psi \in \Gamma$; (b) $U \models (\exists x_{k+1})\psi(\Gamma, x_{k+1})$ iff there is a $\Delta \in E_{k+1}$ such that $\Gamma \subsetneq \Delta$ and $U \models \psi(\Delta)$.

Theorem 1. Let Σ be the set of all sentences of L of length less than C which are true in the standard model; then there is an extension L' of L and a completely finite structure U for L' such that the sentences of Σ are satisfied in U.

By a bounded proof we mean one in which all the formula are of length less than B.

Theorem 2. If Σ is a set of sentences of L from which there is no bounded proof of contradiction then there is a completely finite structure which satisfies Σ; provided that Σ satisfies a (completely finite) condition analogous to 1-consistency.

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Problems in Systems of Mathematical Induction

Fragments of the Arithmetic (systems of open induction) have been studied by many logicians for pure foundational reasons, and have been a source of independence proofs. These theories are based on elementary number theoretic operations, with their recursive definitions as axioms. Mathematical induction schemes for open formulae are added to these systems.

In (2) Shepherdson introduced the theory of Weak Induction which is made up of the following nonlogical operations s, p, +, ., $\dot{-}$, and the following axioms,

A1 $sx \neq 0$

2 $p0=0$

3 $psx=x$

4 $x+0=x$

5 $x+sy=s(x+y)$

6 $x.0=x$

7 $x.sy=x.y+x$

8 $x \dot{-} 0=x$

9 $x \dot{-} sy=p(x \dot{-} y)$

and the axiom of Weak Induction for open formulae with parameters, F,

$$F(0)_\wedge(x)(F(x)\rightarrow F(sx))\rightarrow(x)F(x) \qquad \text{WIO}$$

In my thesis (1) I introduced the weaker theory, Very Weak Induction having the following induction axiom scheme

$$F(0)_\wedge(x)(F(x)\leftrightarrow F(sx))\rightarrow(x)F(x) \qquad \text{VWIO}$$

I gave explicit nonstandard models proving the actual weakness of 'VWIO' relative to 'WIO' (these are the theories whose axioms are composed of A1-9, and the corresponding induction axiom schemes).

The following formulae could be proved using WIO:

B1 $x\neq 0$ $spx=x$

2 $x+y=y+x$

3 $(x+y)+z=x+(y+z)$

4 $x+y=x+z \leftrightarrow y=z$

5 $x.y=y.x$

6 $x.(y.z)=(x.y).z$

7 $x.(y+z)=x.z+y.z$

8 $x \dot{-} y \neq 0 \quad y+(x \dot{-} y)=x$

9 $x \neq y \rightarrow x \dot{-} y \neq 0 \quad y \dot{-} x \neq 0$

10 $x \leq 0 \leftrightarrow x=0$

11 $x \leq sy \leftrightarrow x \leq y \vee x=sy$

12 $x \leq y \rightarrow x+z \leq y+z$

13 $x \leq y \rightarrow x.z \leq y.z$

Where $x \leq y$ iff $y \dot{-} x=0$, and let

C1 $sx \dot{-} x = s0$

C4 $x_1 . y_1 + x_2 . y_2 = x_1 y_2 + x_2 . y_1 \rightarrow x_1 = x_2 \vee y_1 = y_2$

In theorem 1 of (1) I showed that B2-7 are theorems of VWIO

Let the theory 'VWIO' be obtained from 'VWIO' by adjoining B1, B8, C1 and C4. In theorem 3 of (1) I proved that 'WIO' is equivalent to 'VWIO', using model theoretic means. The following problem suggests its self.

Problem 1. Does there exist an algorithm for obtaining a proof of a formula in 'VWIO' once a proof of this formula has been given in 'WIO'. Or equivalently, could one obtain a proof of the inverse of the double arrow in VWIO once the arrow has been proved pointing to the right?

It is worthwhile remarking that for many important formulae the reverse arrow of VWIO follows from pure syntactical symmtries, as seen in (1). Finding an automatic theorem proving device suggests its self.

It is customary in mathematical logic to show the improvability of of a certain formula in a deductive system, by showing that it holds in the standard model while it fails to hold in the nonstandard model.

Problem 2 What does one prove when one proves the independence of a formula in a deductive system using nonstandard models ? More particularly, how could one prove the nonprovability, which is often an infinitistic process,using finitely many steps in the deduction.(such as in the case of the systems of induction)? How much of these systems could be automated?

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PRECIS OF "TOWARD A THEORY OF REVERSIBILITY".

In this paper we develop several ideas –in part stated in our "Reversibilidad en juegos semánticos de conectivas", Actes del V Congrés Català de Lògica (1986), pp. 47-49– related with the concept of reversibility in formal systems and in formal languages.

Due to the noveltry of this approach, as far as we know, we have decided to develop in full lenght a small part of the whole proyect, applied in this case to $\{\vee, \wedge, \neg\}$-languages, instead of trying to give a general account of the topic.

Informally speaking, we understand by the concept of reversibility in a formal system or in a formal language the logical possibility of retracing by means of game-theoretical semantics the process by which we establish in any given system logical properties of its formulae such as universal validity, provability, logical satisfiability and so on.

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Cancellation of lexicographic powers of totally ordered abelian groups

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ABSTRACT:

Let G and H be totally ordered abelian groups and let $G \to H$ denote the lexicographic product of G and H; G^k, the k^{th} lexicographic power of G, is the lexicographic product of k copies of G, $k>0$.

If $G^k \simeq H^k$ (isomorphic as totally ordered groups)

— 1/ If G is such that, for any totally ordered abelian groups A and B:

$G \simeq A \to G \to B$ implies $G \simeq A \to G$

then $G \simeq H$.

Such is the case if G is:

- a/ divisible.
- b/ ω_1 - saturated (applying a result of F. Oger (2)).

(Two totally ordered abelian groups G and H such that $G^2 \simeq H^2$ and $G \not\simeq H$ have been constructed by F. Oger (3)).

— 2/ Also proved by F. Delon and F. Lucas (1) then (letting $\equiv$ denote elementary equivalence) $G \equiv H$. Indeed $G^k \equiv H^k$ and $G \equiv H$ are equivalent.

— 3/ then G and H are isomorphic as chains.

— 4/ If G is such that, for any totally ordered abelian groups A and B:

$G \simeq A \to G \to B$ (as ordered groups) implies $G \simeq_+ A \to G$ (as groups)

then $G \simeq_+ H$ (as groups)

Wether there is any counter example to this is still open.

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(3)- F. Oger: An example of two non isomorphic countable ordered abelian groups with

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Abstract of contributed paper

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Title : Fields with chains : axiomatizations, algebra, 17^{th} Hilbert's problem and Nullstellensatz.

Orderings of higher level on a field have been introduced by Becker (1978) and chains of such orderings by Harman (1982) who also defined chain closed fields. Dickmann proved that these are fields having exactly two orders and Rolle's property for both (to appear).

In a first part (C.R.A.S.,304,1987) we give axiomatizations, in the Artin-Schreier manner, for fields with chains and for chain closed fields ; using axioms of part 1, second and third parts are devoted to chain algebra of commutative rings and of polynomial rings over a chainable field (to appear).

The fourth part (joint work with F. Delon, Sem. D.D.G. 1986-87,Université Paris 7) gives a result of elementary inclusion for some chain closed fields K, and uses it to caracterize elements of $\sum K(X)^{2^n}$. The last part uses part 3 and the elementary inclusion of part 4 to derive a Nullstellensatz over some chain closed fields.

MODAL DEFINABILITY IN ENRICHED LANGUAGES

(Abstract)

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The paper proposed, features a classical algebraic investigation in a promising modal area. The investigation frames the modally definable semantical classes, in the spirit of Goldblatt & Thomason [5, th.3]. The area in question supposes enrichment of the classical modal language. We treat an indicative special case of such an enrichment when besides the classical necessity $\Box$ (with $x \models \Box A$ iff $\forall y(Rxy \rightarrow y \models A)$, the basic language enjoys additionally the "transposed" modality $\boxplus$ with the semantics $x \models \boxplus A$ iff $\forall y(y \models A \rightarrow Rxy)$. This last modality has been consequently rediscovered by many different authors, e.g. van Benthem [1], Humberstone [6]. The last (and most systematic) being the study of Gargov, Passy & Tinchev [3]. They reasonably call this modality "sufficiency", and point out some new perspectives for modal developments of the sort. (In particular applications to modal approximations and expressiveness of first-order phenomena.)

The present paper deals with the algebraic theory of the modal language enriched with "sufficiency" and, consequently with the two "converse" (to $\Box$ and $\boxplus$) modalities from the tense logic, which are quite natural in this context. The modally definable classes of frames for these languages are described. The results considerably differ from the classical case; the reason is that the relevant modal algebras does not form a variety, due to which the classical Birkhoff theorem does not hold any more. The main result about the language with $\Box$ and $\boxplus$ reads (where S-construction is an essential simplification of the SA-construction cf. [5], dual to isomorphic copy of a subalgebra; and the ultraproducts are of full general frames cf. [4])

THEOREM: A class C of Kripke-frames is modally definable iff it is preserved under S-constructions of ultraproducts.

This paper is an initial step towards performing a general modal correspondence theory cf. [2], for interpretations of the predicate calculus into itself.

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THE DEPENDENCE AND INDEPENDENCE OF DEDUCTION RULES IN THE SEMIFORMAL SYSTEM S_2^* OF MARKOV

by

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In 1974, the logician A.A.Markov of the constructive school of Moscow gave a semiformal system of deduction rules for the language $\mathcal{A}_2$, $\mathcal{A}_2$ is one of the extended hierarchy of languages constructed by A.A.Markov in an attempt to formulate his approach to constructive mathematical logic. This system is denoted here by S_2^* and it is different from the systemes S_2 , S_N , $S_{\omega 1}$, and $S_{\omega N}$ of the step - by - step semantic systems studied earlier by the author (Theory of algorithms and mathematical logic. Vy čisi. centre, Akad. Nauk SSSR, Moscow 1974). The semiformal system S_2^* of rules of deduction consists of thirteem rules of deduction DR2.1 - RD2.13.

In this paper, we claim to establish the dependence of each rule RD2.m, where m = 6,7,13, and the independence of each of the rules RD2.n, where n = 3-5,8 - 12. Each rule RD2.m,where m=6,7,13 is dependent in S_2^* in the following sense: For every two closed formulas A and B in the language $\mathcal{A}_2$, if B is deducible from A in S_2^* , then B is deducible from A in $S_2^* \setminus$ RD 2.m. The independence of the rule RD2.n (where n = 3,5,8 - 12) in S_2^* may be proved by finding two closed formulas A and B of the language $\mathcal{A}_2$ such that : B is deducible from A in S_2^* but B is not deducible from A in $S_2^* \setminus$ RD2.n. The independence of the rule RD2.4 in S_2^* may be proved by finding some property P such that each rule from the system $S_2^* \setminus$ RD2.4 preserves P but RD2.4 does not always preserve it.

THE DEVELOPMENT OF PROBABILITY LOGIC - PART I

Theodore Hailperin

The subject was first envisioned by Leibniz: "...I maintain that the study of the degrees of probability would be very valuable and is still lacking, and that this is a serious shortcoming in our treatises of logic. For when we cannot absolutely settle a question one could still establish the degree of likelihood on the evidence, and so one could judge rationally which side is the most plausible."

In making a rational judgement from an argument two aspects are involved: (i) the probability of the chosen premises, and (ii) the manner in which the probability of the premises devolves through the argument to provide a probability for the conclusion. Our study is concerned only with aspect (ii), for we consider determination of the probability of premises, like that of the truth of premises in ordinary logic, to be a non-logical matter.

We appeal to only the simplest formal properties of probability, those which are common to essentially all theories of probability whatever their nature. Conditional probability is also included. The conditional probability of C given A can be interpreted as the probability of (A and A-implies-C), using a renormalized probability measure whose unit is the probability of A.

In this Part I we analyse the contributions to probability logic which occur in the works of Jacob Bernoulli, Lambert, De Morgan, Boole, Peirce, MacColl, and Keynes.

LOGIC PROGRAMMING WITH HIGHER-LEVEL RULES

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We propose a proof-theoretic view of logic programs, according to which a clause of a definite Horn clause program is a rule $A_1,\ldots,A_n \Rightarrow B$ rather than a logically complex formula $(A_1 \& \ldots \& A_n) \supset B$. A program can then be viewed as defining a calculus for the derivation of atomic formulae. A query asks for which joint substitutions certain atomic formulae are derivable in that calculus.

As a generalization of Horn clauses we consider rules of higher levels in the sense of [1], which are obtained by permitting the iteration of the rule arrow "⇒" to the left. A generalized logic program is considered a finite set of higher-level rules. It can again be viewed as defining a calculus, which is now of a natural deduction or sequent type with rules allowed as assumptions. Queries concern the derivability in that calculus. An extended version of SLD-resolution can be defined which is shown to be sound and complete.

In an additional extension, program rules are considered introduction rules for the predicates with which their heads start. Correspondingly, a general elimination schema is added to the formal system defined by the program. In the resulting formalism an "intrinsic" notion of negation is available, the rule of "negation as finite failure" being derivable within the system.

Further generalizations concern the explicit use of quantifiers in higher-level rules as in [2], yielding an evaluation procedure in which substitution of variables is blocked in certain cases.

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NONSTANDARD ANALYSIS FOR THE LAST TWENTY-FIVE YEARS

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ABSTRACT

This is a survey of the development of nonstrandard analysis from 1961 to 1986. It tries to cover the study of nonstandard models and the application of nonstandard analysis to functional analysis, measure theory, probability theory, topology, differential equation, generalized function, and physics. In addition, it involves the study of hyper-natural number system *N and hyper-real number field *R. In the last section some related researches on nonstandard analysis are included.

RESUME

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Equivalence relative to n-ary quantifiers

Generalized quantifiers are understood here in the sense of Lindström (Theoria vol.32 (1966) pp.186-195) except that they are allowed to have infinite types. Thus for any relational vocabulary τ a quantifier Q of type τ is a class of τ-structures closed under isomorphisms. The arity of Q is $\sup\{n_R \mid R\in\tau\}$, where n_R is the arity of R. The semantics of Q is given by the following clause:

$$\mathcal{A}\models Q(x_1^R\ldots x_{n_R}^R\psi_R)_{R\in\tau} \text{ iff } \langle A,(\psi_R^{\mathcal{A}})_{R\in\tau}\rangle\in Q, \text{ where}$$
$$\psi_R^{\mathcal{A}} = \{(a_1,\ldots,a_{n_R})\in A^{n_R} \mid \mathcal{A}\models\psi_R(x_1^R,\ldots,x_{n_R}^R)[a_1,\ldots,a_{n_R}]\}.$$

Let $\mathcal{A}$ and $\mathcal{B}$ be structures of the same vocabulary and $I:\mathcal{A}\sim\mathcal{B}$ a back-and-forth system, i.e. I is a nonempty set of partial isomorphisms from $\mathcal{A}$ to $\mathcal{B}$ with the usual back-and-forth prperties. We say that I is n-bijective ($1\leq n<\omega$) if these properties can be replaced by the following stronger condition:

(*) for each $p\in I$ there exists a bijection $f:A\to B$ such that $p\subseteq f$ and $p\cup f\restriction\{a_1,\ldots,a_n\}\in I$ for all $a_1,\ldots,a_n\in A$.

We write $\mathcal{A}\sim_n\mathcal{B}$ if there exists an n-bijective back-and-forth system $I:\mathcal{A}\sim\mathcal{B}$.

<u>Theorem</u>. $\mathcal{A}\sim_n\mathcal{B}$ iff $\mathcal{A}\equiv_L\mathcal{B}$ for any logic L which is obtained from $L_{\omega\omega}$ by adding n-ary quantifiers.

This theorem has a refinement which deals with quantifier ranks. As an application we prove that for any $1\leq n<\omega$ and $\alpha\in On$ the Magidor-Malitz quantifier Q_α^{n+1} with $\aleph_\alpha$ interpretation is not definable in any logic of the form $L_{\omega\omega}(\{Q_i \mid i\in I\})$, where each Q_i is an n-ary quantifier of finite type.

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Martin"s Axioms, Measurability

and Equiconsistency Results

ABSTRACT

We deal with the consistency strength of ZFC + variants of MA + suitable sets of reals are measurable (and/or Baire, and/or Ramsey). We improve the theorem of Harrington - Shelah [ ] repairing the asymmetry between measure and category, obtaining also the same result for Ramsey. We then prove parallel theorems with weaker versions of Martin's axiom (MA (σ - centered), (MA (σ - linked)), MA ($\Gamma\aleph_0^+$), MA(K)), getting Mahlo, inaccessible and weakly compact cardinals respectively. We prove that if there exists $r \in \mathbb{R}$ such that $\omega_1^{L[r]} = \omega_1$ and MA holds then there exists a Δ_3^1 - selective filter on ω, and from the consistency of ZFC we build a model for ZFC + MA (I) + every Δ_3^1 - set of reals is Lebesgue measurable, has the property of Baire and is Ramsey.

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PROPOSED TALK : Microcontinuity for nonstandard polynomials

S U M M A R Y

For nonstandard polynomials, the monadic concept of microcontinuity is supplemented with the stronger notion of absolute microcontinuity. Relations with the coefficients are established, and it is shown that, whereas microcontinuity may be confined to isolated monads, absolute microcontinuity invariably propagates itself over non-infinitesimal distances. In particular, infinite partial sums of standard power series are examined, revealing a strong intrinsic difference between the notions of convergence (absolute or not) and of microcontinuity (absolute or not).

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ON INTERPRETATIONS OF MANY-SORTED STRUCTURES

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The main notion of this work are interpretations for many-sorted structures. The notion of interpretation in above sense but for the structure with the one universe was introduced in (1). For one-sorted structures it was proved in (2) that any finite composition of elementary interpretations may be represented by a composition of the form QCTE where Q, C, T, E are interpretations defined as: Defining new relation or forgetting about some relation, treating n-tuples of the elements of the old universe as elements of the new universe, restricting of the universe to a definable subset, introducing a new definable equality.

For the many-sorted structures the problem of choice of elementary interpretations to be such generators is more complicated.

The aim of this paper is to show how the choice of generators determines the normal form of the composition of elementary interpretations for many-sorted structures and some other consequences of the above choice.

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A CONTRIBUTION TO THE CONSISTENCY PROBLEM FOR NF

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ABSTRACT

Recent work in fragments of arithmetic ([K,P,D] and [K])suggests that, given an axiom scheme in set theory or arithmetic, by restricting the use of parameters one obtains a new scheme, equiconsistent to the original one, which is not finitely axiomatized (even though the original may have been.) We feel that this is particularly interesting in the case of NF since (i) NF is known to be finitely axiomatizable and (ii) such a "reduced parameter" fragment, NF^-, of NF and equiconsistent with it is presumably not finitely axiomatized (if NF is consistent) and hence we might be able to apply the compactness theorem and other tools from model theory to show NF^- (and therefore also NF itself) has a model. This paper presents the groundwork for such a programme of research. In particular we present several fragments of NF equiconsistent with it and (we conjecture) not finitely axiomatized.

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Full-Homogeneity, How stronger it is comparing to Weak-Homogeneity ?

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In a previous paper [1] we have defined a notion which we called Weak-Homogeneity and proved that if every definable function $f:[\aleph_1]^{\omega^\alpha} \dashrightarrow \aleph_0$ has a weakly-homogeneous set for f, of order type ω^α, then there is a model of ZFC which satisfies $\exists\kappa(o(\kappa)=\alpha)$ and vice versa.

We have also shown that for (Full) Homogeneity the same is true only for $\alpha<3$.

In the present paper we show that Full-Homogeneity is much stronger then Weak-Homogeneity, when dealing with functions on ω^α sequences for $\alpha\geq 3$.
In particular we prove the following,

__Theorem 1:__ If "ZFC + every definable function $f:[\aleph_1]^{\omega^3} \dashrightarrow \aleph_0$ has a homogeneous set for f, of order type ω^3 " is consistent then so is "ZFC + there is a measurable cardinal κ, whose order of measurability $\beta=o(\kappa)$ satisfies $o(\beta)=2$ ".

__Theorem 2:__ If "ZFC + every definable function $f:[\aleph_1]^{\omega^{\omega^\omega}} \dashrightarrow \aleph_0$ has a homogeneous set for f, of order type ω^ω " is consistent then so is "ZFC + there is a measurable cardinal κ, which satisfies $o(\kappa)>\kappa$ ".

Actually this may be not the best possible, since we do not have equiconsistency result yet. For the proofs we define intermediate notions of homogeneity, which may be proved to be useful for other purposes too.

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Gyula Klima: General Terms in Referring Function /Abstract/

Standard quantification theory/SQT/ treats all general terms as predicative expressions. Therefore, in the SQT representation of natural language sentences the referring function of common noun subjects or of complex noun phrases as well as of pronouns with such antecedents is taken over by variables uniformly ranging over the whole universe of a model. This situation, however, gives rise to several difficulties for the SQT representation of natural languages:

1. Formulae of SQT do not preserve the grammatical /NP-VP/ structure of natural language sentences represented by them.
2. SQT breaks radically with the analyses of traditional logic.
3. SQT is incapable of representing a number of quantified natural language sentences /"Most F's are G's", etc./.
4. Despite appearences, the machinery of bound variables of SQT does not properly reflect the workings of pronouns even in cases of simple nominal anaphora. /See problems with "donkey sentences"./

The paper presents a semantic system, GTL, which overcomes these difficulties without departing too far from the usual construction of SQT. The basic idea is the introduction of restricted variables as general terms in referring function. The paper compares GTL with various proposed theories for handling these problems /see references below/ and claims priority to GTL on heuristic grounds.

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INDUCTIVE FULL SATISFACTION CLASSES

Henryk Kotlarski and Zygmunt Ratajczyk

Let Σ_n-PA(S) = PA + S is a full satisfaction class + Σ_n-induction in $L_{PA}\cup\{S\}$. For the definitions see e.g. [1].

We extend the system of ω-logic from [1] in the Turing-Feferman style, i.e. we define formulas Γ^α_r, T^α in L_{PA}. Fix a system of notations for ordinals $<\varepsilon_0$. $T^0(\varphi)$ is "φ is an axiom of PA".

$\Gamma^\alpha_0(\varphi)$ is $T^\alpha \vdash \varphi$

$\Gamma^\alpha_{r+1/2}(\varphi)$ is "φ is of the form $\eta \vee \forall z\, \psi(z)$ and $\forall z\, \Gamma^\alpha_r(\eta \vee \psi(z))$",

$\Gamma^\alpha_{r+1}(\varphi)$ is $T^\alpha \cup \Gamma^\alpha_{r+1/2} \vdash \varphi$

$T^{\alpha+1} = T^\alpha \cup \{\neg\Gamma^\alpha_r(0=1) : r\}$.

For λ limit, let $T^\lambda = \bigcup_{\alpha<\lambda} T^\alpha$.

In the language of $L_{PA}\cup\{S\}$ we define formulas $G_\alpha(x)=y$, which form a hierarchy of quickly growing functions. Let

$G(x) = \min\{y$: whenever the statement $\exists e\, \psi(e,u)$ is in $S \cap \leq x$, there exists e such that the statement $\psi(e,u)$ is in $S\cap \leq y\}$.

The hierarchy G_α is the usual Hardy hierarchy over G, i.e.

$G_0 = id$, $G_{\alpha+1} = G_\alpha \circ G$, $G_\lambda(x) = G_{\{\lambda\}(x)}(x)$ for λ limit.

THEOREM 1 . If a countable, recursively saturated $M \models$ PA satisfies $\neg\Gamma^{\alpha\cdot k}_k(0=1)$ for all standard k, then M has an S so that (M,S) satisfies Δ_0-PA(S) + $\forall a\, \exists G_{\omega\alpha}(a)$.

Let ω_k^c be defined in the usual manner, $\omega_0^c = c$, $\omega_{n+1}^c = \omega^{\omega_n^c}$.

THEOREM 2. If a countable $M \models$PA has an S such that $(M,S)\models \Delta_0$-PA(S)+ $\forall a\, \exists G_{\omega_n^c}(a)$ for all standard c, then M has an S such that $(M,S) \models \Sigma_n$-PA(S).

THEOREM 3. Δ_0-PA(S) + $\forall a\, \exists G_{\omega\alpha}(a) \vdash \neg\Gamma^{\alpha k}_k(0=1)$ for all k.

THEOREM 4. Σ_n-PA(S)$\vdash \forall a\, \exists G_{\omega_n^k}(a)$.

Similar results hold for full PA(S)= $\bigcup_n \Sigma_n$-PA(S). Namely

Theorem 5. A countable recursively saturated $M \models$ PA has an S such that $(M,S) \models$PA(S) iff for each standard n $M \models \neg\Gamma^{\omega_n}_n(0=1)$ iff for each standard n M has an S such that $(M,S) \models \forall a\, \exists G_{\omega_n}(a)$.

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PROPERTIES OF DIAGONALLY NONRECURSIVE FUNCTIONS

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A function f is called diagonally nonrecursive if $f(e) \neq \varphi_e(e)$ for all e. Similarly, a function g is called fixed point free if $W_{g(e)} \neq W_e$ for all e. The class of all (Turing) degrees of diagonally nonrecursive functions coincides with the class of all degrees of fixed point free functions [2]. Let FPF denote this class. Obviously, $\mathbf{0} \notin$ FPF, $\mathbf{0}' \in$ FPF. FPF is meager (in the sense of Baire category) but has Lebesgue measure 1.

The Arslanov completeness criterion [1] shows that $\mathbf{0}'$ is the only r.e. degree in FPF. This criterion was extended in [2] to the REA-hierarchy [3] by showing that if A is n-REA for some n and deg(A) is in FPF then deg(A) $\geq \mathbf{0}'$ (a difference of r.e. sets is a special case of 2-REA level).

There are both global and local interesting subclasses of FPF. Let PA denote the class of degrees of complete extensions of Peano arithmetic. PA coincides with degrees of 0-1 valued diagonally nonrecursive functions ([4] and an unpublished Solovay's result). Degrees in PA cannot be minimal (Scott's and Tennenbaum's result), in fact, such degrees possess various interesting properties ([4], [7]). Another global subclass of FPF is NAP, degrees of NAP sets [5]. NAP sets arise by a diagonalization of Σ^0_1 approximations in measure. The class NAP has Lebesgue measure 1 and no degree in NAP is minimal. There are degrees in NAP − PA below $\mathbf{0}'$ (and, thus, in FPF − PA below $\mathbf{0}'$). NAP sets yield recursively bounded diagonally nonrecursive functions. Nevertheless, it is not known whether all degrees of recursively bounded diagonally nonrecursive functions (or, generally, all degrees in FPF) are not minimal.

An especially interesting local subclass of FPF is the class of degrees in FPF below $\mathbf{0}'$. It can be shown [6] that every degree in FPF which is $\leq \mathbf{0}'$ bounds a nonzero r.e. degree. The method of the proof does not use priority arguments (it has a self-referential flavour). When combined with the low basis theorem [4] it gives an alternative, priority-free, solution to Post's problem (as well as an alternative approach to a considerable part of finite injury priority arguments [6],[7]). This method can be further extended in two ways. In a horizontal direction it can be extended to a larger class of degrees below $\mathbf{0}'$ which arise by a weaker form of a diagonalization. In a vertical direction, leaving the area below $\mathbf{0}'$, the method can be extended in some sense to higher levels of priority arguments. Such extension, however, cannot be done without an additional care since there is a degree in FPF which is r.e. in $\mathbf{0}'$ and bounds no nonzero r.e. degree (hence, too powerful oracle cannot be used straightforwardly). Nevertheless, some extension in this direction is possible. Finally, the basic method (priority-free solution to Post's problem) is provable in the fragment $I\Sigma_1$ of Peano arithmetic due to the fact that the low basis theorem formalizes in this theory (this is a joint result with P. Hájek).

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How Random is a Random Sequence?

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Three different approaches to the definition of random sequences can be distinguished:
- a semi-formal definition by von Mises, based on *admissible place selections*, i.e. a procedure for selection a subsequence of a given sequence x in such a way that the decision to select x_n does not depend on the value of x_n;
- the definition of Martin-Löf using *recursive sequential tests;*
- a definition in terms of (modifications of) Kolmogorov-complexity.

While the relationship between the latter two approaches has been thoroughly explored (e.g. it is known that Martin-Löf randomness can be characterized equivalently in terms of *process complexity* (Scnorr, Levin) or *information* (Chaitin)), the relation between von Mises'' original proposal and Martin-Löf''s definition has received less attention.

Let the measure μ_p on 2^ω be defined by $\mu_p = (1\text{-}p,p)^\omega$, where p is a computable real in (0,1). Let $R(\mu_p)$ denote Martin-Löf's notion of randomness. It is easy to prove that if Φ is a recursive place selection in the sense of Church, then $x \in R(\mu_p)$ implies $\Phi x \in R(\mu_p)$. One feels however, that a random sequence should be much more random than that; not just countably many subsequences should be random w.r.t. the same measure.

Define the operation $/ : 2^\omega \times 2^\omega \to 2^\omega$ by: $(x/y)_m = x_n$ iff n is the index of the m^{th} 1 in y.
We show (see [1],[2])

Thm. *If* $x \in R(\mu_p)$, *then for all non-atomic computable measures* ν: $\nu\{y \in 2^\omega | x/y \in R(\mu_p)\} = 1$.

(The theorem holds as well for the modification of Martin-Löf's definition given by Schnorr.)

A variant of this result, stated in terms of complexity, gives an intuitively satisfying characterisation of the *admissibility* of a place selection.

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(λ^+,λ^+)-COMPACTNESS IMPLIES A WEAK FORM OF (λ,λ)-COMPACTNESS

Paolo Lipparini

Every (λ^+,λ^+)-compact logic allowing relativization is (λ,λ)-compact, provided λ is regular, or $2^\lambda=\lambda^+$. For other λ's we can conclude that L is almost λ^+-(λ,λ)-compact in the following sense: we say that the logic L is almost κ-(λ,μ)-compact iff whenever Σ,Γ are classes of L-sentences, $|\Sigma|\leq\kappa$, $|\Gamma|=\lambda$ and, for every subset Γ' of Γ having cardinality $<\mu$, $\Sigma\cup\Gamma'$ has a model, then there exists a subset Γ° of Γ having cardinality λ such that $\Sigma\cup\Gamma^\circ$ has a model. If the conclusion can be strengthened to: $\Sigma\cup\Gamma$ has a model, we say that L is κ-(λ,μ)-compact. If $\kappa=0$, we omit it.

PROPOSITION. (i) If $\kappa\geq\lambda$ and λ is regular, then almost κ-(λ,λ)-compactness is equivalent to κ-(λ,λ)-compactness.

(ii) Suppose that $\kappa\geq\lambda\geq\nu\geq\mu$ and that there are subsets $(X_\alpha)_{\alpha\in\kappa}$ of λ, each of cardinality ν, such that if $X\subset\lambda$ has cardinality λ, then $X_\alpha\subset X$ [$|X_\alpha\cap X|=\nu$, respectively], for some $\alpha\in\kappa$. (In particular, this holds if $\kappa\geq\lambda^\nu$ [if $\lambda=\nu^+$, respectively])

Then almost κ-(λ,μ)-compactness implies κ-(ν,μ)-compactness. [almost κ-(ν,μ)-compactness, respectively].

(iii) If $\lambda\geq\nu\geq\mu$, then almost κ-(λ,μ)-compactness implies almost κ-(λ,ν)-compactness.

THEOREM. If L is κ-(λ^+,λ^+)-compact and $\nu=\sup\{\kappa,\lambda^+\}$, then either: (i) L is ν-$(\mathrm{cf}\lambda,\mathrm{cf}\lambda)$-compact, and hence (λ^+,λ)-compact, or (ii) for some $\mu<\lambda$ L is almost ν-(λ^+,μ)-compact, and hence almost ν-(λ,λ)-compact.

THEOREM. Suppose that $\lambda>\mathrm{cf}\lambda=\mu$ and that there exist subsets $(X_\alpha)_{\alpha\in\kappa}$ of λ^+, each X_α of cardinality λ, such that for every partition of λ^+ into μ sets, some X_α is contained in the union of $<\mu$ members of the partition. Then κ-(λ^+,λ^+)-compactness implies κ-(λ,λ)-compactness.

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ON QUANTIFIER HIERARCHIES FOR TURING MACHINES

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Alternating Turing machines (ATM) were introduced as a generalization of nondeterministic machines. They turn out very useful in better understanding of many questions in complexity theory and logic.

One of an important problem concerning ATM's is the effect of the number of alternations on the computational power of ATM's. This problem is still unsolved for space- and time-bounded machines and is related with some other long open standing problems in computational complexity and logic. For example, it is well known that the class Δ_o of predicates definable by bounded arithmetic formulas is equal to the class of relations recognized by ATM's working in linear time with constant number of alternations [2] (i.e. $\Delta_o = \bigcup_k \Sigma_k$-TIME (lin)). The problem if the hierarchy Σ_o-TIME (lin) $\subseteq \Sigma_1$-TIME (lin) $\subseteq \ldots$ is proper is strictly related to the problem if $\Delta_o = \varepsilon^2_*$ [3] (ε^2_*- the second Grzegorczyk class [1]) or if Δ_o is equal to the class of relations recognized by deterministic linear space-bounded Turing machines.

Apart from time and space the common complexity measure of Turing machines is the number of reversals which machine head makes during computations (i.e. the number of changes of move direction). For this measure we solve the above mentioned problem.

<u>THEOREM 1.</u> For every well behaved function R and every natural k we have

$$\Sigma_k\text{-REV}(R) \subsetneq \Sigma_{k+1}\text{-REV}(R)$$

where Σ_i-REV(R) denotes the class of languages recognizable by ATM's making at most i alternations and R(n) reversals.

The second result concerns Δ_o relations.

<u>THEOREM 2.</u> Δ_o relations can be recognized by real-time alternating pushdown automata making constant number of alternations and only one reversal on the pushdown store.

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F. LUCAS

On some results of F.Delon and F. Lucas:

CONVEX CLOSURES AND QUOTIENTS OF ORDERED ABELIAN GROUPS.

Peter Schmitt has proved that one can reduce the test for elementary equivalence of two ordered abelian groups G and H , to countably many tests for elementary equivalence between colored chains $Sp_n(G)$ and $Sp_n(H)$ of definable convex subgroups of G and H; in the same way he also gave tests for elementary substru substructures. We use this and well known results of M. Rabin on chains, to investigate some elementary properties of convex closures and quotients of ordered abelian groups.

<u>Theorem 1</u> : If $H \prec G$ are o.a.g. and E is the convex closure of H in G then $H \prec E \prec G$.

<u>Theorem 2</u> : If $H \prec G$ are o.a.g. and H_o is the greatest convex subgroup of G such that $H_o \cap G = \{o\}$, and if for each n, neither $\{o\}$ nor H_o is in $Sp_n(G)$, then $H \prec G/H_o$.

Biblio. : Inclusions et produits de groupes abéliens ordonnés étudiés au premier ordre. (Manuscript). F. Delon and F. Lucas.

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Abstract

SOLO: A second order theory equivalent to Propositional Dynamic Logic.

by María Manzano

Propositional Dynamic Logic is a powerful program logic where most interesting properties of computing programs can be expressed. PDL possesses a weakly complete calculus but it is not a compact logic.

In this paper I present a second order theory, SOLO, which is equivalent to PDL. Three main approaches come together; namely, extended dynamic frames, translating functions and second order axioms.

To define the class E of extended dynamic frames we extend a Kripke PDL model

$$A = \langle W, \langle P_i^A \rangle_{i \in I}, \langle Q_j^A \rangle_{j \in J} \rangle$$

to a second order frame

$$AE = \langle W, W', W'', \langle P_i^A \rangle_{i \in I}, \langle Q_j^A \rangle_{j \in J} \rangle$$

in which sets universes we put in all sets and relations definable in A by PDL formulas and programs.

On the other hand, I have also defined a translating function from PDL formulas and programs to SOL formulas. The idea is to get a SOL formula defining in a frame the set or relation defined in A by the original PDL formula or program. Since we will be using several kinds of frames -i.e., extended dynamic frames, general structures and standard structures- the conclusion changes according to the kind being used.

Finally, I have defined a theory whose axioms are: comprehension axioms for translations of PDL, second order general conditions abstracted from PDL axioms and a very special axiom saying that the loop should be the smallest possible one. As I said before, I have proved that SOLO is equivalent to PDL, both sintactically and semantically.

The aim of this work is to develop a unified theoretical account of the varietv of program logics, second order logic being my choice

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LOGIC COLLOQUIUM '87

Contributed Paper(abstract)

SEMANTIC PROCESSES AND NEGATION

The aim of this paper is to show the relevance of the notion of compositionality in the interpretation of negation as positive informative operation.

To this goal I'll first introduce the concepts of"bottom-up-compositionality"and "top-down-compositionality",by respectively representing them as non logically equivalent semantic processes of construction and de-construction on the subformula structure.

In particular I'll point out that bottom up compositionality as process of semantic construction satisfies the local requirement of informative monotonicity as basic condition, while top down compositionality interpreted as a process of semantic de-construction satisfies the inter-contextual requirement of informative continuity as basic condition.

This distinction between semantic processes will allow a clarification of a further distinction between"Tarskian constituents"and"informative constituents".

On the basis of the given conceptual and notational framework,I'll introduce two not equivalent interpretations of negation;namely:monotonic negation and continuous negation.

Finally,I'll show that:i)any monotonic interpretation of negation is adequate to the bottom up compositional processes of construction;ii)any continuous interpretation of negation is adequate to the top down compositional processes of de-construction.

GENERAL RELATIONAL SEMANTICS FOR ŁUKASIEWICZ'S LOGICS.

by

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Despite the fact that Łukasiewicz systems of many-valued logic were the first non-classical systems investigated in depth, many-valued logic has remained somewhat apart from the mainstream of work in non-classical logics. Thus the semantical methods involving Kripke models seem to have little connection with traditional many-valued logic. In particular this is specially surprising for Łukasiewicz systems, since these methods have proved to be so fruitful in model logic, and Łukasiewicz himself thought of his systems as systems of modal logic.

I will develop a Kripke-style semantics for Łukasiewicz logics, using models of the form (I,f,V), where I is a set, $f: I \rightarrow I$, and $V: P \times I \rightarrow \{1,0\}$ - i.e., V is a two-valued assignement for sentence letters. The basic ideas behind this semantics can be summarized as follows. Let $M = (I,f,V)$, $i \in I$, and let $Ł_m$ be the Łukasiewicz system of m-valued logic.

(1) For any $p \in P$, $(p)^{M,i} =$ Weight(p,s_i), where $(p,s_i) =$ $(V(p,i), V(p,f(i)), V(p,f(f(i))), \ldots V(p,f^1 \ldots f^m(i)))$.

(2) $(Nx)^{M,i} = 1 - (x)^{M,i}$.

(3) $(Cxy)^{M,i} = \min(1,\ 1 - (x)^{M,i} + (y)^{M,i})$.

It can be shown that all Łukasiewicz's logics turn out to be characterized by the class of all ~~models~~ those models.

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Being as Relation

Remarks on the Logic of Plato's Metaphysics

It is an old commonplace in the history of philosophy that Plato's theory of Forms is a suitable explanation for unary predicative statements (or facts) only. His references to this theory in analyzing relational facts are regarded as founded on his alleged ignorance of the difference between characters and relations. H.-N. Castañeda (1) proposed on the basis of some theses in the Phaedo a formal reconstruction of Plato's views regarding relational statements which shows that these views are well considered and coherent, moreover, they can be translated into the language of modern predicate logic. n-ary relational facts involve at Plato not a single n-ary predicate, but a chain of n (unary) Forms. This paper proposes some modifications and an application of Castañeda's reconstruction. At first, it is not necessary to determinate in advance for each Form that it can function only in itself or only as one of an n-membered chain of Forms. There are Forms which things can participate of absolutely (monadically) and relatively (polyadically), too. Secondly, it seems to be not very adequate to Plato's theory to interprete it in an atomistic, set-theoretic semantical framework. We can hope better results from a mereological construction or a mass-term semantic. But this modified Platonic theory of relations can help us to understand Plato's solution to the problem of Non-Being in the Sophistes. By Plato, Being itself is a relation, too, and on this basis he constructs a non-paradoxical reading of statements like 'The Being is not being' or 'The Non-Being is being'.

R e f e r e n c e

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"E-MINGLE HAS THE VARIABLE-SHARING PROPERTY"

E-Mingle (EM) is the result of adding the axiom

(M) $A \rightarrow B \rightarrow . A \rightarrow B \rightarrow . A \rightarrow B$

to Anderson and Belnap's Logic of Entailment, E. As the title makes clear, the aim of this communication is to prove that EM has the variable-sharing property : EM avoids the so-called 'paradoxes of relevance'. In addition, we provide Routley-Meyer type semantics for EM by slightly modifying the standard semantics for E.

Thus, Anderson and Belnap's opinion that "relevance and mingle are incompatible when truth-functions are added" ([1], p. 97) must be qualified. Further, the relationship between 'mere relevance' and 'relevance plus necessity' (entailment) also needs qualifying as shown by the results in [2] upon which we briefly report.

Let R+ be the positive fragment of Anderson and Belnap's Logic of Relevance, R ; let RMO+ be the result of adding the axiom

(M') $A \rightarrow . A \rightarrow A$

to R+. Though, as known, R-Mingle (R plus M') has not the variable-sharing property, we have shown in [2] that either a 'minimal negation' or else a 'semiclassical' one can be added to RMO+ and the resulting system has the variable-sharing property.

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RESTRICTED GENERALIZED QUANTIFIERS

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Let $\boldsymbol{Q}$ denote a family of subsets of a set A. Restricted generalized quantifiers (abbreviated r.g. quantifiers) $\forall_Q$ and $\exists_Q$ are defined as follows. Let L(Q) be a first order language L with additional quantifiers $\forall_Q$ and $\exists_Q$. If **A** is a model of L with domain A and φ is a formula of L(Q), then

$\mathbf{A} \models \forall_Q x\varphi(x)$ iff there is $X \in \boldsymbol{Q}$ such that for all $a \in X$, $\mathbf{A} \models \varphi[a]$.

$\mathbf{A} \models \exists_Q x\varphi(x)$ iff for all $X \in \boldsymbol{Q}$ there is $a \in X$ such that $\mathbf{A} \models \varphi[a]$.

Many generalized quantifiers (as cardinal, filter, topological and measure quantifiers) can be defined using r.g. quantifiers by specifying the choice of family Q:

If $\boldsymbol{Q} = \{A\}$ then $\forall_Q$ and $\exists_Q$ are usual universal and existential quantifiers.

If $\boldsymbol{Q}$ is the family of all nonempty finite subsets of A, then $\forall_Q = \forall$ and $\exists_Q = \exists$.

If $\boldsymbol{Q} = \{X \subseteq A: \text{card}X \geq \omega_1\}$, then $\mathbf{A} \models \forall_Q x\varphi \leftrightarrow Q_1 x\varphi$, $\exists_Q x\varphi \leftrightarrow \neg Q_1 x \neg\varphi x$.

Additional assumptions on family $\boldsymbol{Q}$, give new properties to these quantifiers:

If $\boldsymbol{Q}$ is closed under supersets, then $\mathbf{A} \models \forall_Q x\varphi$ iff $\{a \in A: \mathbf{A} \models \varphi[a]\} \in \boldsymbol{Q}$.

If for all $A, B \in \boldsymbol{Q}$, $A \cap B \neq \emptyset$, then $\mathbf{A} \models \forall_Q x\varphi \rightarrow \exists_Q x\varphi$.

If $B \subseteq A$ and for all $A \in \boldsymbol{Q}$, $A \cap B \neq \emptyset$ implies $B \in \boldsymbol{Q}$, then $\mathbf{A} \models \exists_Q x\varphi \rightarrow \forall_Q x\varphi$. Therefore, if $\boldsymbol{Q}$ is an ultrafilter over A, then $\mathbf{A} \models \exists_Q x\varphi \longleftrightarrow \forall_Q x\varphi$.

If $\boldsymbol{Q}$ is a filter over A, then $\mathbf{A} \models \forall_Q x(\varphi \wedge \psi) \leftrightarrow \forall_Q x\varphi \wedge \forall_Q x\psi$.

We can also define operations and relations over r.g. quantifiers For example, if $\boldsymbol{P}$ and $\boldsymbol{Q}$ are families of subsets of sets A and B respectively, let us define $\boldsymbol{P} \otimes \boldsymbol{Q} = \{X \subseteq A \times B: \{a \in A: \{b \in B: (a,b) \in X\} \in Q\} \in P\}$. Then we study for which families (e.g. the following holds for ultrafilters) we have

$$A \times B \models \forall_P x \forall_Q \varphi(x,y) \longleftrightarrow \forall_{P \otimes Q} z\varphi(\pi_0 z, \pi_1 z), \qquad \forall_{\pi(P \otimes Q)} x\varphi \leftrightarrow \forall_P x\varphi.$$

where π_0 and π_1 are projection functions.

An ordering similar to Rudin-Keisler order for ultrafilters is defined and studied. Finally, the problem for which families $\boldsymbol{Q}$ the completeness theorem for logic L(Q) holds is considered.

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4) $\alpha(v) \vee (v = \overline{\mathrm{Con}_\alpha}) <_{\Sigma_1} \alpha(v) \vee (v = \overline{\neg \mathrm{Con}_\alpha})$, but

$\alpha(v) \vee (v = \overline{\mathrm{Con}_\alpha}) \not\leq_{\Sigma_1 w} \alpha(v) \vee (v = \overline{\neg \mathrm{Con}_\alpha})$

5) If $\beta(v)$ is a "natural" PR binumeration of PRA + "Σ_1 reflection" then, for any consistent Π_1 sentence ϑ,

$\beta(v) \not\leq_{\Sigma_1 w} \alpha(v) \vee (v = \bar{\vartheta})$

6) For any $\beta(v)$, $\delta(v)$, $\beta(v) \vee (v = \overline{\mathrm{Con}_\delta}) \not\leq_{\Sigma_1 w} \delta(v)$

Almost all these results are obtained by means of self-reference.

Self reference and speed up

(by Franco Montagna - Siena)

Let $\alpha(v)$ be a PR binumeration of PRA, E_α be the set of PR binumerations of provable extensions of PRA, Γ be one of Σ_n, Π_n, $(n \geq 1)$. For β, $\delta \in E_\alpha$, define $\beta \leq_\Gamma \delta$ (respectively: $\beta \leq_{\Gamma w} \delta$) iff there is a recursive function (resp. a provably recursive function) $g(z)$ such that, for any Γ- theorem p of PRA, $\mu z\ \mathrm{Proof}_\delta(z,\overline{p}) \leq g \mu z\ \mathrm{Proof}_\beta(z,\overline{p})$. $\leq_\Gamma$ and $\leq_{\Gamma w}$ determine two equivalence relations $\equiv_\Gamma$ and $\equiv_{\Gamma w}$; the corresponding equivalence classes are called Γ degrees (resp. Γw degrees). Call δ a <u>Γ speed up</u> (resp a <u>Γw speed up</u>) of β iff $\beta <_\Gamma \delta$ (resp: iff $\beta <_{\Gamma w} \delta$), $\leq_\Gamma$ has an easy characterization: let $\Gamma(\beta) = \{p \in \Gamma : \mathrm{Pr}_\beta\, \overline{p}\}$; then $\beta \leq_\Gamma \delta$ iff $\Gamma(\beta) \subseteq \Gamma(\delta)$

As regards to $\leq_{\Gamma w}$, we prove:

1) Each Γ w degree is a Σ_2 complete Σ_2 set and contains one element of the form $\alpha(v) \vee (v = \overline{\vartheta}\ )$ (Roughly: a finite extension of PRA).

2) $\leq_{\Gamma w}$ is a lattice preordering.

Another group of results concernes the restriction of $\leq_{\Sigma_1}$ and $\leq_{\Sigma_1 w}$ to extensions of PRA by means of reflection principles.

For example, we prove:

3) $\alpha(v) <_{\Sigma_1 w} \alpha(v) \vee (v = \overline{\mathrm{Con}_\alpha}\ )$ (Roughly: PRA+Con(PRA) is a weak Σ_1 speed up for PRA)

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Pure Logic with Branched Quantifiers

(Abstract)

Branched (or partially ordered) quantifiers were described in several papers (Enderton [1], Wolkoe [2], Mostowski [3]). Let PREF be a set of all branched quantifier prefixes. Let Q ϵ PREF, by L_Q we mean a pure logic with identity (with no non-logical terms) and with Q as an only quantifier. By L_* we mean a pure logic with identity and with all branched prefixes from PREF.

Theorem There are two effective procedures translating of first order formulae into L_*-formulae, $\alpha \mapsto Cons_\alpha$, $\alpha \mapsto Contr_\alpha$ such that for every α: α is consistent if and only if $Cons_\alpha$ is a tautology, and α is contradictory if and only if $Contr_\alpha$ is a tautology.

Corollary Neither the set of all L_*-tautologies nor its complement is recursively enumerable.

Corollary For every finitely axiomatizable theory T there is L_*-sentence β such that T is consistent if and only if β is L_*-tautology. Morever β can be effectively computed from any finite list of axioms of T (We will use ambiguous notation $Cons_T$ for β).

Theorem Let T be selfconsistent (T $\nvdash$ $\neg$CONS(T), where CONS(T) can be taken as described in Smorynski **[4]**.), finitely axiomatizable extension of GB. Then there is finite quantifier prefix Q such that the statement that L_Q is decidable cannot be proved in T. Q can be taken such that $Cons_T$ is L_Q-formula.

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DEFINABLE EXPANSIONS OF MODELS OF PEANO ARITHMETIC

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Let PA be Peano arithmetic and let T denote the second order arithmetic or its fragment. We say that a model M of PA is T-expandable iff there exists a family $\mathcal{X} \subseteq \mathcal{P}(M)$ such that $\langle \mathcal{X}, M, \in \rangle \models T$.

A great deal is known about T-expandable models of PA but still very little is known about concrete T-expansions. In this paper we shall consider a particular construction of expansions of models of PA to models of the theory Δ^1_1-CA + Σ^1_1-AC (it is a fragments of the second order arithmetic consisting of comprehension scheme for Δ^1_1 formulas and the axiom scheme of choice for Σ^1_1 formulas).

Let M be a countable nonstandard model of PA and let S be a full substitutable satisfaction class on M. Let $I \subseteq_e M$ be closed under logical operations. We define the family $\mathrm{Def}_I(M,S)$ of those subsets of M which are definable in M by (possibly nonstandard) formulas from I with parameters from M.

THEOREM 1. Let $I \subseteq_e M$ be closed under logical operations and let $I \neq M$. Then the structure $\langle \mathrm{Def}_I(M,S), M, \in \rangle$ is a model of Δ^1_1-CA + Σ^1_1-AC.

THEOREM 2. Let $I, J \subseteq_e M$ be closed under logical operations and $I \subsetneq J \subsetneq M$. Then $\mathrm{Def}_I(M,S) \subsetneq \mathrm{Def}_J(M,S)$.

THEOREM 3. For any nonstandard countable recursively saturated model $M \models$ PA the family of its countable (Δ^1_1-CA + Σ^1_1-AC)-expansions contains the subfamily of the cardinality of $2^{\aleph_0}$ ordered in the order type of the Cantor set 2^{ω}.

THEOREM 4. For any nonstandard countable recursively saturated model $M \models$ PA there is $2^{\aleph_0}$ full substitutable satisfaction classes on it such that they are inconsistent arbitrarily low.

THEOREM 5. For any nonstandard countable recursively saturated model $M \models$ PA the family of its countable (Δ^1_1-CA + Σ^1_1-AC)-expansions contains a tree of $2^{\aleph_0}$ branches such that each branch of it is of the order type of the Cantor set 2^{ω}.

COROLLARY 6. For any nonstandard countable recursively saturated model $M \models PA$ the family of its countable (Δ^1_1-CA + Σ^1_1-AC)-expansions contains antichains of cardinality $2^{\aleph_0}$.

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Russellian Indefinites, Russellian "Anaphors"

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Russell argued that the logical form of a sentence of the grammatical form 'A ϕ is ψ' can be rendered as $\exists x(\phi x \;\&\; \psi x)$. So-called "discourse anaphora"—as opposed to *bound* anaphora—has been used by philosophers and linguists to undermine this analysis. Strawson and Chastain, for instance, have both objected to a unitary Russellian treatment of indefinite descriptions on the basis of examples like (1) and (2):

(1) Look! A man is eating your turnips; he looks hungry.
(2) A man from China came to see us; he stayed for a week.

If 'he' is anaphoric on the (underlined) indefinite, it is claimed, then the indefinite must be interpreted *referentially*. There are at least two unargued assumptions underlying this claim: (i) the pronoun 'he' refers; (ii) it "picks up" its reference from the indefinite.

Assumption (i) ignores the possibility (suggested by Evans, Cooper, and Parsons) that some pronouns are interpreted as definite descriptions. Assumption (ii) ignores the possibility (suggested by Kripke and Lewis) that some pronouns refer to individuals raised to salience. We argue that these ignored possibilities represent two epistemological sides of a single semantic coin, and propose a theory of pronouns that reflects the epistemological considerations underlying Russell's distinction between singular and general propositions.

The general idea is that pronouns that are "anaphoric" on descriptions, receive either *objectual* or *descriptive* content depending upon epistemological factors. In utterances of (1), 'he' will typically be referential, its *objectual* content attributable to the fact that there will normally be a contextually salient man that can be referred to. In such cases the proposition expressed by 'he looks hungry' will be *singular*. In utterances of (2), 'he' will typically be assigned *descriptive* content—e.g., *the man from China that came to see us*—as an individual will normally have been raised to salience only "under a description." In such cases the proposition expressed by 'he stayed for a week' will be *general*.

A second objection to Russell's theory concerns examples like (3) and (4):

(3) Every man who owns a donkey beats it.
(4) If Pedro owns a donkey he beats it.

As Geach has pointed out, if 'a donkey' is treated existentially and 'it' treated as a variable, the latter will lie outside the scope of the former. Hans Kamp proposes to solve this problem by viewing indefinites as free variables which, roughly speaking, get bound by implicit quantifiers over discourse. This is a clear departure from Russell's proposal, and we show that it is unnecessary if pronouns can have descriptive content. As Benson Mates has emphasized, part of the power of the formal statement of Russell's theory of *definite* descriptions (as per *14.01 and *14.02 of *Principia Mathematica*) lies in the fact that it allows variables within descriptions to be bound by higher operators. On our account, the pronoun 'it' as it occurs in (3) is interpreted as just such a description and hence (3) has the same logical form as 'Every man who owns a donkey beats the donkey he owns', which can be represented as:

$$\forall x((\text{man}(x) \;\&\; \exists y(\text{donkey}(y) \;\&\; \text{owns}(x,y))) \rightarrow \text{beats}(x, (\iota z)(\text{donkey}(z) \;\&\; \text{owns}(x,z)))).$$

We conclude by defusing objections to the implication of relative uniqueness in the above.

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Resumen del trabajo: "Sobre la posibilidad de una lógica ilocutiva"

Presentado por: Prof. Susana Nuccetelli (Universidad Autónoma de Madrid)

La propuesta de una lógica ilocutiva (J.Searle y D.Vanderveken) se asienta sobre nociones controvertidas. Sus conceptos básicos de 'acto ilocutivo' y 'fuerza ilocutiva' no están exentos de objeciones. La forma lógica del acto de habla elemental que evitaría las inconsistencias de Austin sería F(P), donde 'F' está por la fuerza ilocutiva y 'P' por el contenido o acto proposicional . Sin embargo, hay teorías alternativas, que adoptan diferentes actitudes frente a los conceptos de Austin. Sus propuestas son:

1-mantener la distinción locutivo/ilocutivo introduciendo modificaciones en la teoría de Austin. Se puede admitir que parte de la fuerza ilocutiva está en el acto locutivo, como lo muestran los controvertidos ejemplos de Austin. Se podría suprimir el recurso al estilo indirecto como criterio para diferenciar actos réticos de ilocutivos. El ámbito de aplicación de valores de verdad sería un tipo de actos ilocutivos, los enunciados.

2-reformular la distinción locutivo/ilocutivo manteniendo la noción de fuerza ilocutiva como parte del significado de una emisión. La relación de la fuerza ilocutiva con otros aspectos del significado plantea cuestiones interesantes para la semántica, como la existente entre el prescriptivismo y el cognitivismo.

3-eliminar la distinción locutivo/ilocutivo y también a las fuerzas ilocutivas. Es suficiente el concepto fregueano de sentido para explicar lo que Austin pretende.

La posición eliminativista ha ganado terreno en las semánticas de la lengua natural. De hecho, no se ha requerido un aparato lógico especial para dar cuenta de las fuerzas ilocutivas, ya que mediante performativos explícitos o la descomposición y paráfrasis indicativa puede transformarse el componente no indicativo. Estos enfoques cuestionan la necesidad de una lógica ilocutiva.

Nuccetelli

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GENERALIZED INDUCTION AND UNCOUNTABLE GAMES

by

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We use uncountable games to generalize ordinary inductive definability. Let Γ be a monotone operator on a set X and Γ^∞ be the set inductively defined by Γ. The relation $x \in \Gamma^\infty$ has the following well-known characterization. Imagine a game between two players E and A, A choosing elements x_i from X (with $x_o = x$) and E replying with subsets X_i of X. Player E wins, if $x_{i+1} \in X_i \to x_{i+1} \in \Gamma(X_{i+1})$ for all i, and $X_i = \phi$ for some $i \in \omega$. Now $x \in \Gamma^\infty$ if and only if player E has a winning strategy in this game.

We describe a framework in which it is meaningful to consider a similar game of the length of an arbitrary ordinal. This game leads to our generalization of inductive definability. Our games are not necessarily determined as opposed to the above games of length ω. Accordingly we have two different generalizations of inductive definability. In the first we ask whether E has a winning strategy and in the second we ask whether A does not have one.

Our results include versions of standard results of the theory of inductive definability such as the Stage Comparison Theorem and the Kleene Theorem. This work is closely related to and was originally motivated by a project in which model theory for infinite quantifier languages is developed.

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A LOGICAL ANALYSIS OF TERM REWRITE SYSTEMS AND COMPUTATION OF EQUATIONAL THEORIES

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Term rewrite systems are logic-free equational systems without the symmetric axiom (the axiom of the form a=b => b=a). Then equality a=b has an oriented direction and is interpreted a reduction or a transformation from one term to another. We use the expression s->t for a reduction of a term s to a term t. By a conditional equational system we mean a system of a finite set of conditional equations of the following form;

$$u_1=v_1, u_2=v_2, \dots, u_n=v_n \Rightarrow l=r,$$

where ui, vj, l and r are terms, and => is the symbol for conditional. We assume a fixed set of function symbols and constant symbols. By a conditional term rewrite system we mean a system of a finite set of conditional rules of the following form:

$$u_1|v_1, u_2|v_2, \dots, u_n|v_n \Rightarrow l\to r,$$

where l->r means the reduction of a term l to a term r. u|v mean "u and v are joinable", i.e., "there is a term w such that u->*w and v->*w". Here ->* is a finite sequence of reductions, i.e., the transitive and reflexive closure of ->. Transitivity and reflexivity correspond to the transitivity axiom and the reflexivity axiom of equational systems, respectively. The system is said to be left-linear if l of each rule has only one occurrence for each variable.

Our goal is to obtain criteria of equivalence between a given conditional equational system and the corresponding conditional rewrite system, or to make a conditional rewrite system which is equivalent to a given conditional equational system. Here equivalence between a conditional equational system E and a conditional rewrite system R means "E |- s=t if and only if R|- s|t". Equality s=t is interpreted in a rewrite system as $s|r_1|r_2|\dots|r_n|t$ for some terms $r_1, r_2, \dots, r_n$. The reduction of s=t to s|t means a direct computation of s=t. A rewrite system is said to be terminating if there is no infinite sequence of reductions. A rewrite system is said to be confluent if, for any term t, s, u such that t*<-u->*s, t|s holds. A rewrite system is said to be locally confluent if, for any term t, s, u such that t<-u->s, t|s holds. If a rewrite system is terminating and confluent, then the system is equivalent to the corresponding equational system. The condition "confluent" can be replaced by "locally confluent", because termination and local confluence imply confluence. Reductions s->t and u->v are "overlapping" if a subterm of s is unifiable with u or a subterm of u is unifiable with s. If reductions s->t and u->v are overlapping, the most general unification r for s and u is called the overlap. The pair p and q obtained from the overlap r by applications of reductions s->t and u->v respectively is called a critical pair. It is known (by Dershowitz and Sivakumar) that if a terminating conditional rewrite system is non-overlapping, the system is confluent.

THEOREM1. There exists a terminating conditional rewrite system in which every critical pair are joinable but which is not confluent.

DEFINITION. (1) An overlapping of two terms s,t is called an overlay or a top-overlap if s and t share the same root at the overlap, i.e., s and t overlap at the same the outermost function symbols .If every overlapping is overlay, then we call the system an overlay system. (2) (depth of a proof). 2.1 The depth of a proof of s->t is 0 if s->t is the result of an application of a non-conditional rule (of the form => l->r). 2.2 The depth of a proof of s->t is k+1 where k is the maximum number of depths of subproofs for conditions $u_1|v_1, \dots, u_n|v_n$ if s->t is the result of an application of a conditional rule which has a substitution instance of the form $u_1|v_1, \dots, u_n|v_n \Rightarrow l\to r$. 2.3 The depth of a proof of $s\to s_1\to s_2\to\dots\to s_m\to v\leftarrow t_n\leftarrow\dots\leftarrow t_2\leftarrow t_1\leftarrow t$ is the maximum number of depths of subproofs for $s\to s_1, s_1\to s_2, \dots, s_m\to v, t_n\to v, \dots, t_1\to t_2, t\to t_1$. (3) (a normal proof and a normal depth). 3.1 A normal proof of s->t is s->t itself if s->t is the result of an application of a non-conditional rule (of the form => l->r). 3.2 A normal proof of s->t consists of normal proofs for conditions $u_1|v_1, \dots, u_n|v_n$ and a substitution instance $u_1|v_1, \dots, u_n|v_n \Rightarrow l\to r$ of a conditional rule if s->t is the result of an application of the rule. 3.3 A normal proof of s|t is a proof of the form $s\to s_1\to s_2\to\dots\to s_m\to v\leftarrow t_n\leftarrow\dots\leftarrow t_2\leftarrow t_1\leftarrow t$ for a normal form v and normal proofs for $s\to s_1, s_1\to s_2, \dots, s_m\to v, t_n\to v, \dots, t_1\to t_2, t\to t_1$. 3.4 A normal depth of s->t or s|t is a depth of a normal proof for s->t or s|t respectively. (4) (immediately joinable) A critical pair s,t are said to be "immediately joinable" if s->t or t->s holds or there is u such that s->u<-t. (5) (shallow joinable) 5.1 For a critical pair s,t and the overlap u of the form s<-u->t such that u->s has the depth n and u->t has the depth m, the critical pair are "shallow joinable" if there exists v such that t->*v is provable with depth less than or equal to n and s->*v is provable with depth less than or equal to m. 5.2 For a critical pair s,t and the overlap u of the form s<-u->t such that u->s has the normal depth n and u->t has the normal depth m, the critical pair are "normally shallow joinable" if there exists a normal form (i.e., irreducible form) v such that t->*v is provable with normal depth less than or equal to n and s->*v is provable with normal depth less than or equal to m. (6) (normal system) 6.1 For a normal form (i.e., irreducible form) N and a term s, a condition of the form s|N is called a "normal condition" or a "Bergstra-Klop condition". 6.2 A conditional rewrite system in which every conditional rule is of the form $s_1|N_1, \dots, s_n|N_n \Rightarrow r\to l$ for normal conditions $s_i|N_i$ is called a "normal conditional system".

THEOREM2. (1)(Okada-Sivakumar) Every terminating overlay conditional rewrite system is confluent. (2) For any left-linear normal conditional system, if every critical pair are shallow joinable then the system is locally confluent. Hence if the system is terminating, then it is confluent. (3) For any left-linear normal conditional system, if every critical pair are shallow joinable and immediately joinable, then the system is confluent. (4) For any left-linear system, if every critical pair are normally shallow joinable, then the system is locally confluent. If the system is terminating, it is confluent. (5) The form corresponding to (3) also holds.)

On the other hand, (2) cannot be improved by removing one of the conditions "left-linear" or "normal system".

THE RANK OF $\mathcal{C}$-HARDY FIELDS IN SEVERAL VARIABLES

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In [3], [4] we have defined, utilizing the notion of filter $\mathfrak{F}$ of subsets of $\overline{\mathbb{R}^n}$ converging to $\bar{0} \in \overline{\mathbb{R}^n}$, the $\mathcal{C}$-fields and the $\mathcal{C}$-Hardy fields in several variables. $\overline{\mathbb{R}^n}$ denotes any one-point compactification of the euclidean n-space $\mathbb{R}^n$ to a point $\perp \notin \mathbb{R}^n$. Moreover $\mathcal{C}$ is any smoothness category of real valued functions in n real variables [2]. Denoting by K any field as above we can state the following results and definitions [5].

<u>Proposition</u> - The function $p : K \to \mathbb{R} \cup \{\pm\infty\}$ defined by $p(f) = \lim_{\mathfrak{F}} f(\underline{x})$ is a signed place.

<u>Definition</u> - The rank of K is the rank of the valuation υ generated over K by the place p.

Moreover following [7], by results of [3], we can define

<u>Definition</u> - The rational rank of K is the dimension of the vector subspace over Q generated by $\upsilon(K^*)$ in $\upsilon(K'^*)$ where K' is the real closure of K.

Giving then, for all $n \in \mathbb{N}$, a manner to construct filters $\mathfrak{F}_n$ over $\overline{\mathbb{R}^n}$ with basis f formed by semi-algebraic cells of $\mathbb{R}^n$ [8], we prove:

<u>Theorem</u> - The ring K_n of germs in 0 following $\mathfrak{F}_n$ of n-variables rational functions is a Nash-Hardy field. Its real closure, $\bar{K}_n$, is the Hardy field of germs of n-variables Nash functions. Moreover rank K_n = rank $\bar{K}_n = n$.

In [6] we give examples of Pfaff-Hardy fields in several variables with infinity rank. In fact here, utilizing the notion of Pfaff function [1], we study the existence of exponentially and logaritmically closed Hardy fields in several variables. These arguments seem to have some interesting connection with the O-minimal theories extending Th($\mathbb{R}$) formulated in a finitary first order language which extends the language $\{+, \cdot, -, 0, I, <\}$ for the ordered rings.

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Searching games with constant error probability.

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Abstract:

We consider the following two-person searching game on the set $\{1,\ldots,n\}$. The Responder thinks of a number $x \in \{1,\ldots,n\}$, unknown to the Questioner who has to find x by asking questions $x < a$?, for $a \in \{1,\ldots,n\}$. The Responder answers each query before the next question is asked but he lies with known probability $0 < p < 1$, for each answer independently. The result of the Questioner's search has to be correct with a given reliability $0 < q < 1$.

We show that this search is feasible for any reliability $0 < q < 1$ if and only if $p \neq \frac{1}{2}$. For any such p the Questioner's winning strategy of length $O(\log^2 n)$ is given. For $0 < p < \frac{1}{3}$ or $\frac{2}{3} < p < 1$ we show a strategy of length $O(\log n)$.

A NOTE ON APPROXIMATE COUNTING

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In many combinatorial problems we have to count objects having a certain property. Abstractly it can be described as a problem of computing for a given set $A \subseteq \Sigma^*$, $\Sigma = \{0,1\}$ values of function $c_A(n) = |A^n|$, where $A^n = A \cap \Sigma^n$. The corresponding desision problem is to recognize for any A (maybe from a certain class) a language $COUNT(A) = graph(c_A)$, where $graph(f) = \{1^n \# k \mid k = f(n)\}$.

A natural upper bound on Turing machine computation complexity of this problem is LINSPACE $\subseteq$ POLSPACE. On the other hand, it has been proved that there is a set A for which PARITY problem (i.e. recognizing $graph(c_A \bmod 2)$) does not belong to any level of the polynomial-time hierarchy.

Sometimes it suffices to compute an approximation of c_A. Stockmeyer has proved that for any constant $r > 1$ there exists Δ_3^p-machine which for any set A given as its oracle computes an approximation of c_A within the factor r. We recall that $g: N \to N$ approximates $f: N \to N$ within a factor $r > 1$ if

$$f(n)/r \leqslant g(n) \leqslant rf(n) \quad \text{for any } n \in N.$$

He has also proved a corresponding lower bound: there is a recursive set A for which any $\Delta_2^p(A)$-machine cannot compute a function which approximates c_A within a constant factor.

We improved this lower bound proving:

Theorem There is a recursive oracle A such that for any constant $r > 1$

$$ACOUNT_*(A,r) \cap \Sigma_2^p(A) = \emptyset,$$

where $ACOUNT_*(A,r) = \{graph(f) \mid f \text{ approximates } c_A \text{ within the constant } r\}$.

UNIVERSITAT DE BARCELONA
FACULTAT DE MATEMÀTIQUES

Barcelona, 28-III-1987

Le envío el "abstract" de mi contribución:

LA TEORIA DE CONJUNTS A DEDEKIND
(LA TEORÍA DE CONJUNTOS EN DEDEKIND)
(THE DEDEKIND's SET THEORY)

L'article analitza el desenvolupament de la teoria de conjunts en DEDEKIND tot analitzant una dotzena dels seus treballs.

El artículo analiza el desarrollo de la teoría de conjuntos en DEDEKIND, basándose en la lectura de una docena de sus trabajos.

This paper analyse the progress of the DEDEKIND's set theory, founding it on the lecture of a dozen of its works.

Atentamente,

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THE CRAIG INTERPOLATION LEMMA FOR QUANTIFIED MODAL LOGICS

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Let Φ be the class of schemes which are built of variables α_i by means of connectives $\Box$, $\wedge$; Ψ - the class of schemes which are built of $\Box^n \alpha_i$ by means of $\wedge$, $\vee$. Let Ξ be the class that contains Φ and is closed under $\Box$, $\wedge$ and under the following rule:

if $\psi \in \Psi$, $\xi \in \Xi$ and any variable α_i occuring positively in ξ does not occur negatively twice (or more times) in $\psi \to \xi$ then $\psi \to \xi \in \Xi$.

Let $\mathcal{L}$ be the class of K-normal propositional logics which are axiomatized by some schemes from Ξ . Any logic $L \in \mathcal{L}$ is known to be sound and complete with respect to the class of Kripke models in which the accessibility relation satisfies some conditions $R(L)$. By the counterpart $L1$ of L we understand the first order modal logic determined by the class of Kripke models with nested domains which satisfy $R(L)$. Complete axiomatization of $L1$ contains the one of L , classical quantifier rules and usually also some additional axioms.

We present a complete axiomatization of any logic $L1$ where $L \in \mathcal{L}$ and we prove the usual versions of the Craig, Robinson and Beth theorems for these logics. One can see that the Barcan formula is not a theorem of any of the logics. The paper contains also similar results for some K-regular logics and for some Segerberg-complete logics which do not contain the formula K. (We hope that till July the classes of considered logics will be essentially enlarged.)

The method of the proof is based on a classical interpretation of modal theories and on a representation of modal algebras. The method does not depend on any deduction lemma.

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An axiomatization of Fuzzy Set Theory

Various axiomatizations of Fuzzy Set Theory (FST) have been proposed, [C], [W], [N], having in common the fact of having a ternary predicate $\in(x,y,z)$. Even if they formalize Zadeh's idea of Fuzzy Sets, from different points of view they seem to be not suited to axiomatize the intuitive idea of vague (fuzzy) objects, while, for example, the valuation object of a vague class is itself a vague object (in everyday language) and can change from a class to another class.

[N] is not sufficientely developped to be judged, while the other ones fix (in some sense) a valuation object once for all, and put a relation on the valuation object that is well-defined (non-fuzzy) so that the valuation object is not completely fuzzy.

If we want to axiomatize fuzzy objects in which the valuation object can change, we must assume a membership relation represented by a quaternary predicate $\in(x,y,z,v)$, and then we are able to build a theory that parallels the theory of Kelley-Morse.

In the theory we are not able to define equality on all objects due to the choice of having valuation objects that are as fuzzy as the starting objects, in any case we are able to set in the theory an axiom of construction by which we are able to define a unique intersection, between objects and a unique union and so on, as it seems to be expected from the everyday language.

Crisp objects are properly defined in our theory and it can be shown that the theory is coherent with Kelley- Morse Set Theory

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BASES FOR THE CLOSED UNBOUNDED FILTER.

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ABSTRACT: We consider subfilters of the closed unbounded filter on $P_\kappa(\lambda)$ for κ a regular uncountable cardinal and $\lambda > \kappa$. Under certain hypothesis on λ, for each $n \in \omega$, $n > 1$, there is a filter F_n on $P_\kappa(\lambda)$ such that $F_n \subsetneqq [\Delta F_n] \subsetneqq \ldots \subsetneqq [\Delta^{n-1} F_n] \subsetneqq [\Delta^n F_n] = \text{CLUB}$.

APPROXIMATION LOGIC

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ABSTRACT

In many branches of computer science there is a need of approximation methods. We present an approximation logic as a tool for investigating different approximation problems in computer science.

The first step to construct an approximation logic was given in [5]. In that paper the rough concepts logic based on the first order predicate calculus was formulated.

We extend the rough concepts logic to a family of approximation logics by introducing decreasing sequences (finite or infinite of order $\omega + 1$) of indiscernibility relations and various kinds of approximation operators determined by these relations. All considered operators have properties of the modal operators in $S5$ modal logic [1]. If the sequence of indiscernibility relations is infinite then the $(\omega + 1)$-th indiscernibility relation is the intersecion of relations in that sequence.

An axiomatisation for that type of approximation logics is given and the theorem establishing the completeness of that axiomatisation is proved.

The notions of approximating algorithm and its semantics are crucial steps in construction of our approximation algorithmic logic. Many problems of algorithm approximation are expressible in that logic. We investigate also the problem of approximating translation.

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Constructive uniform proofs on finite structures.

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Abstract: The uniform proofs that we study are associated to inductive definitions on classes of finite structures [1, 2, 4]. To an inductive definition of a predicate P on a class **K**, corresponds a *uniform proof* of P, i.e. a function which for each structure U of **K** defines a proof of P in **U**.

Inductive definitions distinguish recursion variables from parameters, and this distinction is used in a fundamental way in the calculus of inductive queries [3]. This allows to represent a class of algorithms defining a predicate P, for which we associate a uniform proof of P. We describe the constructivity of these proofs, that use computations on sets and the finiteness of the structures in a fundamental way, and show their non-monotonicity. As examples, we describe classical shortest paths algorithms as inductive definitions on the class of finite labeled graphs, and their corresponding proofs.

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Jaballa Salah

"Complexité du calcul du développement d'un nombre réel en fractions continues"

Abstract :

In this note, we are concerned with the use of continued fraction representation of real numbers in the formal computationnel theory of recursive analysis. In the recursive case, the most general representation for real numbers is the Cauchy sequence representation. On one hand, other representations such as the Dedekind cut representation and the binary expansions have been studied and shown not to be as general as the Cauchy sequence representation. On the other hand, the class of real numbers with primitive recursive continued fractions is identical with the class of primitive recursive real numbers in the sense of Cauchy which are recursively irrational. Furthermore, there is no efficient algorithm for implementing addition on reals numbers written in the continued fraction form.

1. Introduction :

La notion de réel récursif, c'est-à-dire calculable de façon effective par un algorithme, est connue depuis Turing [12] et a été largement étudiée par Specker [11]. Plus récemment, K.I. Ko [4] a introduit dans la théorie des outils modernes de complexité et en particulier la notion de temps polynômial. Cependant, tous ces auteurs ont privilégié l'étude des suites convergentes récursives (primitives récursives, polynômialement calculables) ou des développements dyadiques récursifs (primitifs récursifs, polynômiaux), mais se sont peu intéressés au point de vue des fractions continues.

Dans cet article, nous étudions cette notion à partir du développement d'un nombre réel en fraction continue ; un nombre réel récursif en ce sens étant un réel tel que la suite de ses quotients partiels d'ordre n est récursive.

Nous démontrons que si toutes ces définitions sont équivalentes pour la notion de réel récursif, il n'en est pas de même si l'on s'intéresse aux notions de réel primitif recursif ou, par exemple, de réel calculable en temps polynômial.

Le résultat principal est une caractérisation des nombres réels dont le développement en fractions continues est primitif récursif comme ceux qui sont :

- limites d'une suite primitive récursive (p.r.)
- p.r. irrationnels (ce qui exprime l'existence d'une fonction p.r. et telle que : $\forall m \; \forall n \quad |x - \frac{m}{n} > \frac{1}{r(n)}$).

Une conséquence de ce résultat est une formulation rigoureuse du fait qu'il n'y a pas de 'calcul simple' du développement d'une somme à partir des développements de chacun des termes.

CONTINUOUS FUNCTIONS : l-GROUPS AND DECISION PROBLEMS

Shen Fuxing* Volker Weispfenning**

For any topological space I, we let C(I) (C*(I)) denote the set of all (bounded) continuous real-valued functions on I. The algebraic and model theoretic properties of C(I) and C*(I) as rings have been studied by many authors. Cherlin has shown that for most spaces I, Th C(I) is undecidable. We study the first-order properties of C(I) and C*(I) as lattice-ordered additive groups (l-groups).

THEOREM 1 There is a primitive recursive procedure assigning to any elementary statement p about C(I) or C*(I) as l-groups an elementary statement p' about lattice Z(I) of zero-sets in I, such that p and p' are equivalent for all T_4-spaces I.

COROLLARY 1 If I and J are T_4-spaces such that Z(I) and Z(J) are elementarily equivalent as lattices, then C*(I), C*(J), C(I) and C(J) are elementariy equivalent as l-groups.

Using a famous result of Rabin's, we get:

COROLLARY 2 The elementary theory of C(I) and C*(I) as l-groups is primitive recurcively decidable for I = R or I = 2^{N}.

Let R(I) be the boolean algebra of regular open subsets of I.

THEOREM 2 Suppose B = R(I) and I is metric or B = Z(I) and I is an open subspace of the real line. Then there is a polynomial time procedure assigning to every lattice-statement p about B an l-group statement p' about C(I) or C*(I) such that p and p' are equivalent.

COROLLARY 3 Let I,J be open subspace of the real line R. Then C(I) and C(J) are isomorphic as l-groups iff I and J are homeomorphic.

COROLLARY 4 For any non-compact subspace I of the real line R, C(I) and C*(I) are elementariry inequivalent as rings.

COROLLARY 5 The decision problem for the elementary theory of C(R) is not elementary recurcive.

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TRANSFORMATIONS AND DEVELOPMENT OF LOGIC PROGRAMS

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We say that a logic program A' extends a logic program A if for every predicate symbol p in the language of A, there is a predicate symbol p' of A' and terms $s_1, \ldots, s_k$ such that for every appropriate tuple of terms t_i,

$$A \text{ solves the goal } p(t_1,\ldots) \text{ iff } A' \text{ solves } p'(t_1,\ldots,s_1,\ldots,s_k)$$

We say that the goal $p'(t_1,\ldots,s_1,\ldots,s_k)$ corresponds to the goal $p(t_1,\ldots)$. Note that the terms s_i depend only on p and not on t_j's. Hence, A' need not compute exactly the same relations as A, however, every relation computed by A is definable as a subset of a projection of a relation computed by A'.

Using transformations defined in [1], we can prove the following

<u>Theorem</u> For every logic program A, there is an extension A' such that A' has a computation tree with

(i) at most one OR-node called by recursion,

(ii) at most one alternation of AND- and OR-nodes on every branch.

Moreover, the corresponding goals are computed by the same number of steps by both programs.

The transformations used in the proof of the above theorem can be used in development of logic programs, hovever, they are conservative, i.e. they preserve the number of steps for every goal. We are going to describe another type of transformations, which depend on the specific properties of relations computed by the program and give programs with more effective computations.

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Towards a Theory of Evidential Reasoning in Logic Programming

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ABSTRACT

Van Emden [VE86] has given a logical framework for quantitative reasoning in logic programming. In this paper, we extend the proposals of Van Emden and that of [S87] to give a logical basis for *evidential reasoning* in logic programming. A logic programming language is proposed and an *evidential logic program* (ELP) is defined to be a finite set of universally closed formulas of the form

$$L_0 : [\mu_0, \nu_0] \Leftarrow L_1 : [\mu_1, \nu_1] \& \ldots \& L_k : [\mu_k, \nu_k]$$

where each L_i is a literal, $k \geq 0$, and each $\mu_i, \nu_i \in [0,1]$. An infinite valued *paraconsistent* logic in which the truth values are members of $[0,1] \times [0,1]$ is proposed and the semantics of ELPs is defined in terms of this logic. The models of an ELP E are characterized in terms of the pre-fixpoints of a monotone operator T_E from Herbrand interpretations to Herbrand interpretations. It is shown that the least fixpoint of T_E is computable and is exactly the same as $T_E \uparrow \omega$ (defined in a similar way as in [LL84]). We also give an operational semantics for ELPs. We define a model $\mathcal{M}$ of an ELP E to be *nice* iff for every atom A in the Herbrand base B_E of E , $\mathcal{M}(A) = [\mu, \nu]$ and $\mu + \nu \leq 1$. In addition, we identify a distinguished recursive subset of the class of all ELPs called *well-behaved* ELPs and show that they possess nice models. It is proved that the set of nice, supported [ABW86] models of a well-behaved ELP has both a least and a greatest element, and that they are both computable and identical to $T_E \uparrow \omega$ and $T_E \downarrow \omega$ respectively. An outline of an SLD-resolution-like operational semantics is given and soundness and weak completeness results are derived. A quantitative characterization of the *degree of inconsistency* and/or the *degree of under-determinedness* of an atom $A \in B_E$ with respect to the least model of the ELP E is obtained.

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A CHARACTERIZATION OF A CLASS OF REGULAR MODAL LOGICS /ABSTRACT/

A modal logic L is called normal iff $\vdash_L \Box(A \wedge B) \Leftrightarrow \Box A \wedge \Box B$ and L is closed under Rule of Necessitation. The smallest normal modal logic is denoted by K. A modal logic L is said to be regular iff $\vdash_L \Box(A \wedge B) \Leftrightarrow \Box A \wedge \Box B$ and L is closed under Regularity Rule. The smallest regular modal logic is denoted by C2. A modal logic L is called strictly regular iff L is regular and is not normal.

Let $\underline{2}^-$ denote the modal algebra $\langle \{0,1\}, \cup, \cap, -, N \rangle$ where $N1 = N0 = 0$ and let $L(\underline{2}^-)$ be modal logic characterized by the algebra $\underline{2}^-$. We have

THEOREM 1. Let L be a normal modal logic. Then

$$\vdash_L A \quad \text{iff} \quad \vdash_{L \cap L(\underline{2}^-)} \Box\top \rightarrow A .$$

THEOREM 2. For a strictly regular modal logic L the following conditions are equivalent

(i) $L = M \cap L(\underline{2}^-)$ for a normal modal logic M,

(ii) $(\Box\top \rightarrow \Box\Box\top) \in L$.

Let $C2.1. = C2 \cup \{\Box\top \rightarrow \Box\Box\top\}$ and let $\Lambda(C2.1.)$ denote the lattice of strictly regular modal logics extending C2.1. Let $\Lambda(K)$ denote the lattice of normal modal logics.

THEOREM 3. Let $f: \Lambda(K) \longrightarrow \Lambda(C2.1)$ be a function defined by the equality $f(L) = L \cap L(\underline{2}^-)$. Then f establishes an isomorphism between $\Lambda(K)$ and $\Lambda(C2.1.)$

Theorems 1 and 2 are extensions of some theorems from [1]

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ABSTRACT

Recursive formalisms and arithmetic

A recursive formalism (R.F.) is a collection of formal systems based on the propositional calculus with a common language, all of them finitarily or constructively consistent. The way to use a R.F. in the development of a theory has to be strictly constructive. The members of a R.F. $\mathcal{F}$ are named segments of $\mathcal{F}$, and denoted $\bar{V}, \bar{W}, \ldots$.

$\bar{U} < \bar{V}$ means that every axiom of $\bar{U}$ is axiom of $\bar{V}$.

$\bar{V} : \mathcal{A}$ means "$\mathcal{A}$ is true in the segment $\bar{V}$".

For example, if we use $\exists$ with its intuitive constructive meaning, then $\bar{V} : (\exists \underline{x}) [\vdash \underline{A}(\underline{x})]$ means "we can exhibit a term $\underline{t}$ for which we can prove $\bar{V} \vdash \underline{A}(\underline{t})$".

A new symbol $\Rightarrow$ must be introduced with the following meaning:

$\bar{V} : \mathcal{A} \Rightarrow \mathcal{B}$ (read as "$\mathcal{A} \Rightarrow \mathcal{B}$ is true in $\bar{V}$") means "if $\bar{V} : \mathcal{A}$, then we can exhibit some other segment $\bar{W}$ such that $\bar{V} < \bar{W}$ and $\bar{W} : \mathcal{B}$".

$\mathcal{F} : \mathcal{C}$ means "$\bar{V} : \mathcal{C}$ for every segment $\bar{V}$ of $\mathcal{F}$". In particular, $\mathcal{C}$ may be of the form $\mathcal{A} \Rightarrow \mathcal{B}$.

The development of a theory by means of a R.F. $\mathcal{F}$ consists in proving constructively some assertions of the kind $\mathcal{F} : \mathcal{C}$, where the $\mathcal{C}$ are some assertions interpretable or meaningful in any segment of $\mathcal{F}$.

A R.F. named "formally recursive arithmetic" is defined. It is a formalism stronger than recursive arithmetic, almost as strong as classical arithmetic. Based on it, a "constructively consistent mathematical analysis" may be developped, not having the most important limitations of constructive analysis.

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About the completeness of lattice valued logics

Abstract

Logic is conceived as an ordered pair $(\mathcal{L}, Cal)$ where $\mathcal{L}$ - the language - is an ordered quadruple $(\mathbb{M}, \mathbb{F}, \mathbb{T}, \models)$ where $\mathbb{M}$ is the class of the models in the classical sense; $\mathbb{F}$ is a formal language - being built up according to the conventialal rules for generating well-formed formulas - with the help of which one can say something about the members of a model from $\mathbb{M}$; $\mathbb{T}$ is a structure of the truth values, in the present case an arbitrary lattice and $\models$ is the truth evaluation function mapping $\mathbb{M} \times \mathbb{F}$ into $\mathbb{T}$. Cal is an algorithm called calculus, in its intuitive meaning, which effectivizes a partial function mapping $\omega \times 2^{\mathbb{F}}$ into $\mathbb{F}$ thus enumerating a subset of the model theoretic consequences of a given subset of sentences. There is shown that in the case of a complete lattice $\mathbb{T}$ there exists a sound calculus and that the compactness theorem holds. Introducing a generalized completeness concept, the corresponding generalized completeness theorem is proved.

ProTalk : a Prolog-based interactive interpreter for Knowledge Acquisition

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A Prolog-based programming-through-dialogue environment, ProTalk, is presented which tries to deal with some of the problems arising in the common border among Knowledge Representation, Knowledge Acquisition and Man-Machine Interaction.

The aim of this system is that of creating a powerful environment for Knowledge Acquisition through *the achievement of simmetry ir. man-machine communication*. Its main component is a metalevel interpreter which embodies the feature of EQtU (Extended Query-the-User), an improvement of Marek Sergot's Query-the-User.

With EQtU any distinction between programming and use is abolished.

The paper will show how in ProTalk any action that can be performed by either man or machine can also be performed by the other one, thus reaching a simmetric, dialogic interaction and a friendly environment for Knowledge Acquisition.

During the evaluation of a query,the system asks (EQtUs) the user for any missing information, and the user may reply :

- by instantiating variables
- by telling ProTalk any number of facts or rules
- by asking some other queries, in order to decide what to do
- by making a hypothesis, that is giving facts and rules considered to be enough to solve the question.

If the hypothesis should not suffice, the user will face another question and will decide between giving futher information or letting the query fail. In the latter case all hypothesized clauses (facts or rules) will disappear from the DB. Any number of EQtUs can take place during the evaluation of a single query, so that the user can incrementally build up large programs without fear to forget any particular which would lead Prolog to the criptical "no" answer.

ProTalk can handle several distinct theories (knowledge bases), at one time, and switch from one to another, achieving full nonmonotonicity.

Open or closed-world assumtion can be specified for a theory when it is created, and theories can be created or destroyed dinamically.

The paper will also describe the "why" and "how" explanations the system embodies, giving the user a way to check system's deductions.

The paper will also clearly show how all of ProTalk 's facilities can fit in the unifying framework of man-machine simmetry. Only symbolic variable names are used throughout, instead of Prolog's awful "_XYZ" ones.

The paper will also describe how partial evaluation helps gain efficiency, and the consistency checks carried out within each theory.

Linguistically Definable Concepts and Dependencies

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Abstract

1 Linguistically Definable Concepts

The subject of machine learning (see [Michalski, Carbonell, Mitchell 1983], [Bar, Reingenbaum 1981]) has received considerable attention in recent years and different authors ([Banerji 1980], [Quinlan 1983], [Mitchell 1982]) provide different models for its description. We will use here a semantic model for inductive reasoning ([Orłowska 1986]) which the schemes of inductive reasoning cover those domains which can be describrd by the listing of the following conceptual primitives: object, attribute, and value of the attribute. Objects are caracterized by means of attributes which are meaningful for these objects, every attribute can take on values from a fixed set of values and we assume that those sets of objects, attributes, and value of the attributes are fixed for a given domain i.e that by a universe of discourse we mean a system $U = (OB, AT, VAL)$ where OB is a non-empty set of objects, AT is a non-empty set of attributes, and, we have for each attribute $a \in AT$ a set VAL_a associated with it. VAL is the union of all the sets VAL_a. Given a universe U, by information about an object we mean the describtion of the object in terms of the values of all its attributes i.e by an information we will understand a function $f : OB \times AT \to VAL$ such that $f(o, a) \in VAL_a$ for all $o \in OB$ and $a \in AT$. Elementary knowledge about objects from a given universe is provided by information about values of attributes for the objects and we can consider similarity, or indiscernibility of objects with respect to some attributes. Usually this knowledge is expressed by means of a certain condition expressed in terms of information about objects. We will assume hence, that we are given a subset A of the set AT and that we are interested in some conditions $C_R(f(o_1, a), ..., f(o_n, a))$ for $a \in A$ and $n \geq 2$. We define a family of relations $R(A)$ in set OB as follows:

$$(o_1, ..., o_n) \in R(A) \text{ iff } C_R(f(o_1, a), ..., f(o_n, a)) \text{ holds for all } a \in A.$$

These relations are called em associations between objects and they reflect our *background knowledge* about objects.

Now we can consider more complex elements of knowledge, namely *concepts.* In its semantic interpretation, a concept is represented as a subset of a universe of discourse. Given a concept, we want to characterise it taking into account background knowledge represented by the associations admitted for the given universe.

We will deal here only with associations which are reflexive, transitive and symetric i.e which are represented by a certain equivalence relation, but our consideration can be ganeralized to the class of at least reflexive relations, as it is defined in [Orłowska 1986].

We say that a concept C represented semantically in U by a set $X \subset OB$ is $R(A)$ *definable* if X can be represented as the union of some equivalence classes of the relation $R(A)$.

We say that a concept C represented semantically in U by a set $X \subset OB$ is $R(A)$ *linguistically definable* if a context-free grammar can be constructed in such a way that we can define a function ψ, called the interpretation of $L(G)$ in X, such that $X = \psi(L(G))$.

The following is true.

Fact 1 *Any $R(A)$ definable concept is $R(A)$ linguistically definable.*

We can define, as in [Orłowska 1986] upper $\overline{R(A)}X$ and lower $\underline{R(A)}X$ approximations of X representing the concept **C**. Intuitively, the lower approximation of a set X consists of those objects which definitively, according to knowledge represented by $R(A)$, belong to X and the upper approximation consists of those objects which possibly, according to knowledge $R(A)$ belong to X.

We say that a concept **C** represented semantically in U by a set $X \subset OB$ is $\overline{R(A)}$ ($\underline{R(A)}$) definable if $X = \overline{R(A)}(Y)$ ($X = \underline{R(A)}(Y)$) for a certain set $Y \subset OB$.

- $\overline{R(A)}$ ($\underline{R(A)}$) definable concepts seems to be also lingiustically definable, but the proof of it is an open problem.

2 Linguistically Definable Dependencies

[1]

Functional and multifunctional dependencies are important and widely studied integrity constraints for relational databases ([Sagiv,Fagin 1981], [Commadakis, Kanellakis, Spyratos 1986]). They can be expressed in terms the indiscernibility relations. In [Rauszer 1986] has shown that a dependency holds in a database if and only if a corresponding algebraic formula is equal to the greatest element in the algebra. The indiscernibility relation is defined intuitively as follows: if X is a subset of the set of all attributes of a relation **R**, then two tuples are in indiscernibility relation, denoted by X^*, provided they have the same X-value in **R**. The dependencies can be expressed in terms of indiscernibility relations as follows. Let X, Y be sets of attributes of a relation **R** over a set of attributes U and FD, MVD denote functional and multifunctional dependencies, respectively.

$$FD X \rightarrow Y \text{ holds in } \mathbf{R} \text{ iff } X^* \subseteq Y^*.$$
$$MVD X \rightarrow\rightarrow Y \text{ holds in } \mathbf{R} \text{ iff } X^* \subseteq (X \cup Y)^* \circ (X \cup (U - Y))^*.$$

With each functional and multifunctional dependency D we can associate a formula α_D of a proper FD or MVD logic.

We say that the dependency D is *linguistically definable* if there is a grammar **G** such that the following is true. The dependency D holds iff $\alpha_D \in L(\mathbf{G})$.

We prove that

Fact 2 *Both, functional and multifunctional dependencies are linguistically definable.*

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